Using Splines to Characterize Unmodeled Systematic Errors

Herman L. Marshall
(MIT Kavli Institute)
Previous Presentations

- **IACHEC1**: Goal is to avoid two problems
  - A: claims of new physics due to calibration errors
  - B: features ignored due to presumed systematics
- **IACHEC2**: Two new tools
  - Multiple adjustment functions (HLM) — bad
  - Vary instrument models (Drake et al.) — good
- **IACHEC3**: Update
  - Dewey’s “science relevance” $\chi^2/\nu$ adjustment
  - More of Drake’s method
- **IACHEC4**: no update
OLD Adjustment Method

- Method proposed: Use penalty function
  - Minimize
    \[ \Lambda = \sum_{j} A_j^2 + \exp\left(\frac{(\chi_{\nu j}^2 - 1)^2}{\nu_j}\right) \]
  - where
    \[ \chi_{\nu n}^2 = \frac{1}{\nu_n} \sum_{i=1}^{I_n} \left[ y_{in} - f(x_{in}; \bar{\alpha})(1 + \sum_{j} A_{nj}g(x_{in}; \beta_j)) \right]^2 \]

- Problems:
  - min \(\chi^2/\nu\) achieved jointly: 2.62, 1.48
  - Model is “ugly”

- Solutions?
  - Different basis functions
  - Evolve toward Drake et al. method
Example: Mk 421 LETGS

- Gaussian normalizations computed
- Not consistent between LETGS observations
Example: Mk 421 LETGS

Mk 421 LETGS Oct 2002, LETGS, with CNOF edges

- $N_H$: $1.2000000e+20$
- $\tau_{C-K}$: $-0.0075387371$
- $\tau_{N-K}$: $-0.10812451$
- $\tau_{O-K}$: $-0.080659293$
- $\tau_{F-K}$: $-0.048417862$
- $\tau_{F-K}$: $-0.086276533$
- $A_1$: $0.91264050$
- $\Gamma_1$: $2.1657286$
- $A_2$: $0.87997260$
- $\Gamma_2$: $1.0385369$
NEW Adjustment Method

- Method: spline amplitudes
  - Define correction grid (wavelength, energy, ...)
  - Correction amplitudes defined on grid (init = 0)
  - Adjust $A_{\text{eff}}$ by spline through amplitudes
  - Creates a smooth adjustment with arbitrary shape

- Method succeeds at a “reasonable” level

- Future:
  - Examine distribution of amplitudes
  - Centroids of amplitudes --> fix EA
  - Standard deviations --> characterization of errors
Mk 421 LETGS again

Mk 421 LETGS Oct 2002, LETGS, with CNOF edges

\[ N_H: \quad 1.2000000e+20 \]
\[ A_1: \quad 0.96659040 \]
\[ \Gamma_1: \quad 2.1957328 \]
\[ A_2: \quad 0.99050049 \]
\[ \Gamma_2: \quad 1.2332592 \]

\[ \tau_{C-K} \quad -0.025648936 \]
\[ \tau_{N-K} \quad -0.070360124 \]
\[ \tau_{O-K} \quad -0.029455254 \]
\[ \tau_{F-K} \quad -0.023914065 \]
\[ \tau_{F-K} \quad -0.073464241 \]

Energy (keV)
Normalizations

Spline amplitudes $\sim$ Gaussian norms
Results at a Glance

Mk 421 Oct. 2002

Spline model

Frac. Residuals

Wavelength (Å)

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HLM — Systematic Errors
Are We There Yet?

Predicted Gaussian

Residual (σ) vs. N
Are We There Yet?
Are We There Yet?

Broadened Gaussian

![Graph showing a broadened Gaussian distribution with residuals on the x-axis and counts on the y-axis.]

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Are We There Yet?

![Broadened Gaussian](image)

Not quite
Apply to new data

Before

Mk 421 LETGS March 2010, LETGS, with CNOF edges

Flux (ph/cm²/s/keV)

1.00
0.10
0.01

Energy (keV)

10

τ_{C-K} = 0.63566439
τ_{N-K} = 0.0000000
τ_{O-K} = 3.6663602e-17
τ_{F-K} = 1.5265234e-143

N_{H} = 1.200000e+20
A_{1} = 1.0666900
A_{2} = 2.4920410
A_{2} = 0.61514652
Γ_{2} = 1.5653655
Apply to new data

Mk 421 LETGS March 2010, LETGS, with CNOF edges

\[\begin{align*}
\tau_{c-K} & = 0.63566439 \\
\tau_{N-K} & = 0.0000000 \\
\tau_{O-K} & = 3.6663602 \times 10^{-17} \\
\tau_{F-K} & = 1.5265234 \times 10^{-143}
\end{align*}\]

\[\begin{align*}
N_h & = 1.2000000 \times 10^{20} \\
A_1 & = 1.0666900 \\
\Gamma_1 & = 2.4920410 \\
A_2 & = 0.61514652 \\
\Gamma_2 & = 1.5553655
\end{align*}\]
Apply to new data

After

HLM — Systematic Errors
Before

\[ \chi^2 = 2.81 \]
After
Before

After

\[ \chi^2 = 1.82 \]
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