Characterizing Systematic Errors

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Previous Presentations

IACHEC1: Goal is to avoid two problems
- A: claims of new physics due to calibration errors
- B: features ignored due to presumed systematics

IACHEC2: Two new tools
- Multiple adjustment functions (HLM) — bad
- Vary instrument models (Drake et al.) — good

IACHEC3: Update
- Dewey’s “science relevance” $\chi^2/\nu$ adjustment
- More of Drake’s method

IACHEC4: no update

IACHEC5: Use of splines for adjustments
Spline Adjustment Method

- Method: spline amplitudes
  - Define correction grid (wavelength, energy, ...)
  - Correction amplitudes defined on grid (init = 0)
  - Adjust $A_{eff}$ by spline through amplitudes
  - Creates a smooth adjustment with arbitrary shape

- Use:
  - Characterizing systematic errors
  - Distribution of examples of systematic errors
  - Informing calibration scientists to fix problems
Mk 421 LETGS

Flux (ph/cm$^2$/s/keV)

Energy (keV)

$N_H$: $1.2000000 \times 10^{20}$

$A_1$: 0.96659040

$\Gamma_1$: 2.1957328

$A_2$: 0.99050049

$\Gamma_2$: 1.2332592

$\tau_{C-K}$: -0.025648936

$\tau_{N-K}$: -0.070360124

$\tau_{O-K}$: -0.029455254

$\tau_{F-K}$: -0.023914065

$\tau_{F-K}$: -0.073464241
Normalizations

Spline amplitudes ~ Gaussian norms
Results at a Glance
Are We There Yet?

Predicted Gaussian

N

100.0

10.0

1.0

0.1

-10

-5

0

5

10

Residual (σ)
Are We There Yet?
Are We There Yet?

Broadened Gaussian
Are We There Yet?

Broadened Gaussian

Not quite

Residual (σ)
Apply to new data

Before

Mk 421 LETGS March 2010, LETGS, with CNOF edges

\[ N_h : 1.2000000 \times 10^{20} \]
\[ A_1 : 1.0666900 \]
\[ A_2 : 2.4920410 \]
\[ A_3 : 0.61514652 \]
\[ \Gamma_2 : 1.3653655 \]

\[ \tau_{c-K} : 0.63566439 \]
\[ \tau_{N-K} : 0.0000000 \]
\[ \tau_{O-K} : 3.6663602 \times 10^{-17} \]
\[ \tau_{F-K} : 1.5265234 \times 10^{-143} \]

Flux (ph/cm²/s/keV)

Energy (keV)
Apply to new data
Apply to new data

After

Mk 421 LETGS March 2010, LETGS, with CNOF edges

\[ \begin{align*}
N_h: & \quad 1.2000000e+20 \\
A_1: & \quad 1.0666900 \\
\Gamma_1: & \quad 2.4920410 \\
A_2: & \quad 0.61514652 \\
\Gamma_2: & \quad 1.5553655 \\
\end{align*} \]
Before

\[ \chi^2 = 2.81 \]
Before

After
$\chi^2 = 1.82$
Spline Adjustment Method

- Method: spline amplitudes
  - Define correction grid (wavelength, energy, ...)
  - Correction amplitudes defined on grid (init = 0)
  - Adjust $A_{\text{eff}}$ by spline through amplitudes
  - Creates a smooth adjustment with arbitrary shape
- Method succeeds at a “reasonable” level
- A use of method:
  - Make spline EA model for xspec & isis
  - Publish “candidate” adjustment amplitudes
  - Collect users’ fit results
  - Use amplitudes as input to
Fitting Power Laws in Narrow Energy Ranges

Objective: Coarse characterization of systematic errors

Method (see M. Smith’s presentation):
- Define narrow energy bands
- Fit power law to spectrum in each band
- Compute flux in each band using model
- Compute confidence interval for each flux
- Compare fluxes for different instruments

Claim: flux is robust to error in model

Concern: RMFs require spectrum outside band
Application to Chandra

- Cross-check results with direct measurement
- Data = \{C_i, E_i\}, measured in time T
- Effective area = A_i
- Default estimator:

\[ F(E_1, E_2) = \sum_{E_i = E_1}^{E_i = E_2} \frac{C_i E_i}{tA_i} \]
Consider simple case

- Source has invariant photon flux $n$
- Observe twice with effective area $A$
- Exposure times are $t_1, t_2$, counts $C_1, C_2$

One estimate of $n$:

$$n = \frac{n_1 / \sigma_1^2 + n_2 / \sigma_1^2}{1 / \sigma_1^2 + 2 / \sigma_1^2}, n_1 = \frac{C_1}{t_1 A}, n_2 = \frac{C_2}{t_2 A}, \sigma_1 = \frac{\sqrt{C_1}}{t_1 A}, \sigma_2 = \frac{\sqrt{C_2}}{t_2 A}$$

ML estimate of $n$:

$$n = \frac{C_1 + C_2}{A(t_1 + t_2)}$$
Diversion 2

Case 2: two observations, different areas

\[ n = \frac{C_1+C_2}{A_1 t_1 + A_2 t_2} \]

Case 3: estimate energy flux, F

\[ F = \frac{C_1+C_2}{A_1 t_1 / E_1 + A_2 t_2 / E_2} \]

Case 4: estimate flux if \( n(E) = K (E/\hat{E})^{-\Gamma} \)

\[ F = \frac{\hat{E}(C_1+C_2)}{A_1 t_1 (E_1 / \hat{E})^{-\Gamma} + A_2 t_2 (E_2 / \hat{E})^{-\Gamma}} \]
Central Energy

Model: \( 1 + \log\left(\frac{E_2}{E_1}\right)^2 \frac{(2 - \Gamma)}{24} \)
Flux Sensitivity

The graph shows the flux sensitivity as a function of \( \Gamma \) for different values of the parameter, indicated by the lines: 0.33 - 0.54, 0.54 - 0.85, 0.85 - 1.50, 1.50 - 4.00, and 4.00 - 10.00. The x-axis represents \( \Gamma \) ranging from -2 to 4, and the y-axis represents \( \frac{F}{F(\Gamma=1)} \) ranging from 0.96 to 1.10.
Summary of Bandpass Fitting

- Simple to do, can get “acceptable” fits
- Flux is robust to knowledge of spectral slope
  - \(\rightarrow\) provides easy measure for cross calibration
- Requires knowledge of spectral slope
  - can estimate from data in band
  - however, slope changes slowly \(\rightarrow\) use wide band
- Need to include case of wide RMFs