Xiao-Li Meng
Under the guidance of Vinay Kashyap (and Herman Marshall & Matteo Guainazzi)

A Log-Normal Linear Regression Approach for (Simultaneously) Adjusting Attributes

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A problem posed by Dr. Matteo Guainazzi

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Systematic errors in comparing effective areas:
Speaking hypothetically, if we label the instruments by numbers $i = 1, \ldots, N$ and each has an attribute A that is used to measure the same $j = 1, \ldots, M$ astrophysical sources, with intrinsic attribute F_j where $C_{ij} = A_i F_j$ are the instrumental measurements, then the question is: "Is there a way to decide how (or whether) to change A_i when the values C_{ij}/A_i do not agree with F_j to within their statistical uncertainties s_i?" In other words, each instrument provides an estimator f_j of F_j with statistical uncertainty s_j but $|f_j - F_j|/s_j$ is often large, not distributed as a Gaussian with unit variance (but can have zero mean if we define $F_j = \sum_j f_j s_j^{-2} / \sum_j s_j^{-2}$). How to estimate the systematic error on the A_i?

Instruments (i) and Sources (j)

- i are <u>individual detectors</u> (e.g., *Chandra*/ACIS-I, *Chandra*/HEG, *XMM*/EPIC-pn, *XMM*/EPIC-MOS1, *XMM*/RGS2, *Swift*/XRT, *Suzaku*/XIS, *NuSTAR*/FPMA, *Integral*/ISGRI, etc.), <u>with counts obtained in specific passbands</u> (e.g., soft=[0.5-2 keV], hard=[2-7 keV], ultra=[10-30 keV], etc.)

- j are <u>individual sources</u> (HZ 43, Capella, PKS 2155-304, Mkn 421, Crab, G21.5-09, etc.) <u>with fluxes predicted in specific passbands</u>

From Vinay ...

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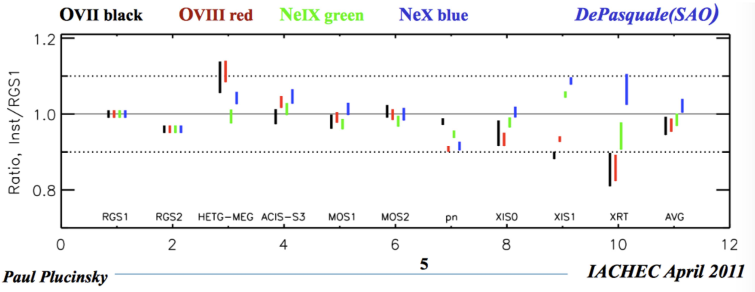

i ≡ [RGS1, RGS2, HETG-MEG, ACIS-S3, MOS1, MOS2, pn, XIS0, XIS1, XRT] **x** [560-574 eV, 654 eV, 905-922 eV, 1022 eV] (i=1..10,11..20,21..30,31..40)

j ≡ E0102 fluxes in [OVII, OVIII, NeIX, NeX] (j=1..4)

- $c_{1,1}$ = observed counts in RGS2/[560-574 eV], $c_{12,2}$ = in HETG-MEG/[654 eV], $c_{23,3}$ = in ACIS-S3/[905-922 eV], etc.
- a_i = effective area, f_j = expected flux, α_{ij} = exposure time of instrument i for source j (in this case, $\alpha_{k(\cdot)}$ are identical for $k=\{l, l+10, l+20, l+30\}$, l=1..10)

Avoid mixing (deterministic) estimands with (random) estimators

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- Let A_i be the *actual* effective area of instrument i; F_j be the *true* flux of source j; then the *expected* rate can be modelled as

$$C_{ij} = A_i F_j \quad \text{or} \quad \log C_{ij} = \log A_i + \log F_j$$

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- Let a_i be an estimator of A_i; f_j an estimator of F_j, and c_{ij} be the actual observation from source j detected by instrument i.
- Then it is *NOT* reasonable to expect $c_{ij} \approx a_i f_j$, in the sense of justifying the "regression" model

$$\log c_{ij} = \log a_i + \log f_j + e_{ij}, \quad E(e_{ij}) = 0.$$

Distributions cannot be manipulated as numbers

For (deterministic) numbers Y and X

$$\textit{If} \quad Y = \rho X, \quad \text{then} \quad X = \rho^{-1} Y.$$

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If regressing Y on X yields (both have zero mean and unit var):

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Then regressing X on Y is NOT $X = \rho^{-1} Y$, but rather

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Here ρ is the *correlation* between X and Y .

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- Do not follow "The Rule of Three" (Stephen Stigler, *Seven Pillars of Statistics*; ASA President Address, 2014).

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$$y_{ij} = \alpha_{ij} + B_i + G_j + e_{ij}$$

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- set $\eta_j = 0$ when we do not want to adjust f_j.

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Therefore, when we set

$$E(y_{ij}) = -0.5\sigma_{ij}^2 + B_i + G_j,$$

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When variances are known, simply "correct" the data

$$y'_{ij} = y_{ij} + 0.5\sigma_{ij}^2; \quad g'_i = g_i + 0.5\tau_i^2; \quad f'_j = f_j + 0.5\eta_j^2$$

Shrinkage estimators: combining information

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Two sources of information (assuming model (I))

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  average of $\{y_{ij}' - g_j, j \in J_i\}$, M_i the size of J_i.

- **Relative precision:** $w_i = \tau_i^{-2}/(\tau_i^{-2} + M_i\sigma_i^{-2})$

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Maximum Likelihood Estimation: Linear shrinkage on log-scale

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- We assume the error for g_j is negligible; $\eta_j^2 = 0$

Power shrinkage on the original scale

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Using A=exp(B), we obtain the adjustment

$$\hat{A}_i = \left[a_i e^{\tau_i^2/2}\right]^{w_i} \left[(\tilde{c}_{i\cdot}\tilde{f}_i^{-1}) e^{\sigma_i^2/2}\right]^{(1-w_i)}$$

where $\tilde{c}_{i\cdot}$ and $\tilde{f}_i$ are the geometric means:

$$\tilde{c}_{i\cdot} = \left[\prod_{j \in J_i} c_{ij}\right]^{1/M_i}, \quad \tilde{f}_i = \left[\prod_{j \in J_i} f_j\right]^{1/M_i}$$

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- When $\sigma_i^2/M_i << \tau_i^2$, $w_i \to 0$, suggesting we need more and more adjustment.

If this is all non-sense, hope you are

as jet-lagged as me!