Another way to analyze IXPE data using XSPEC

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[Some review of CalStats presentation on July 1, 2022 at https://iachec.org/calibration-statistics/]

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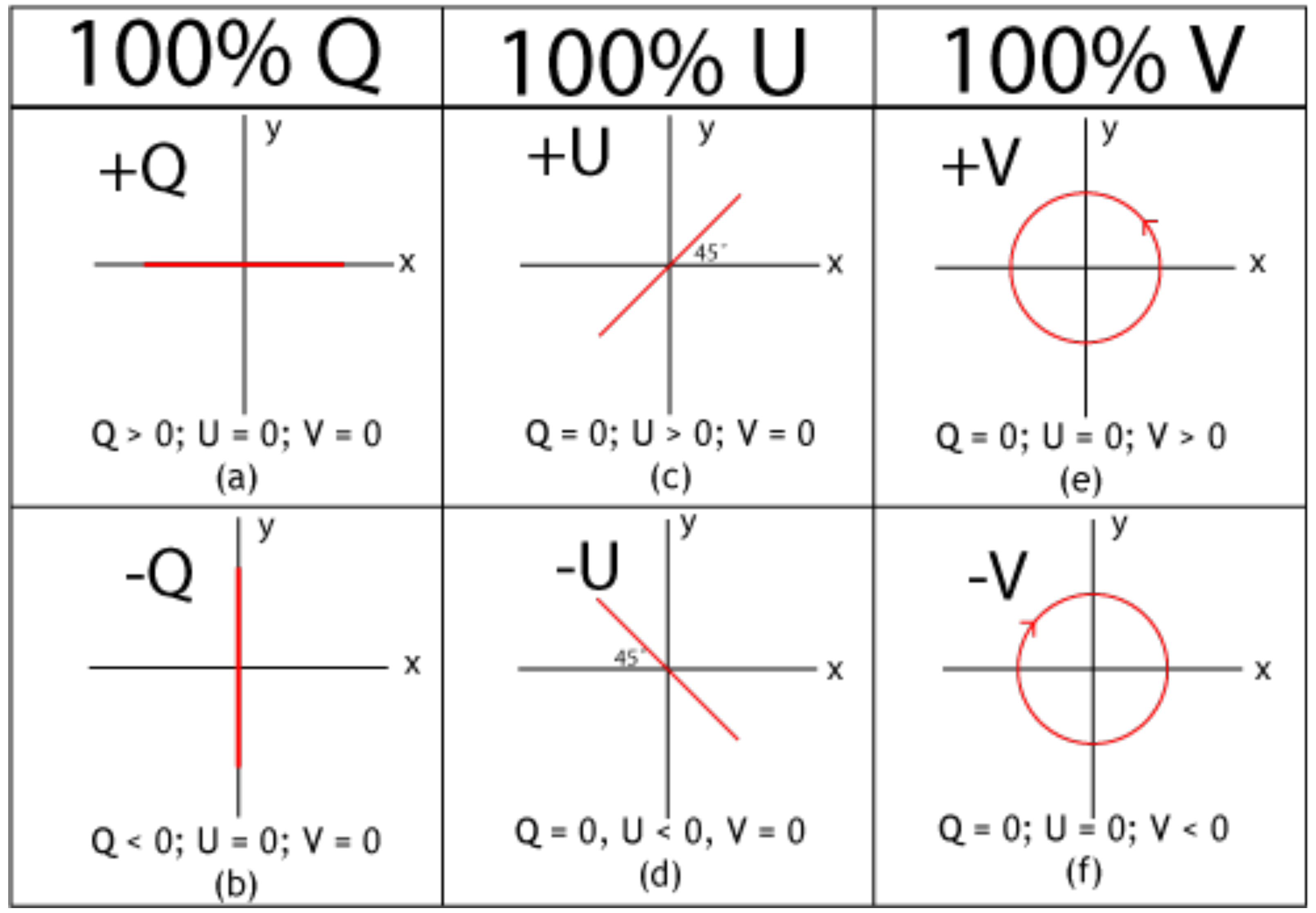
A Few Basics

- Stokes parameters are handy:
  - I = total intensity
  - Q, U are orthogonal linearly polarized parts
  - V is circular (+ or -) polarized intensity

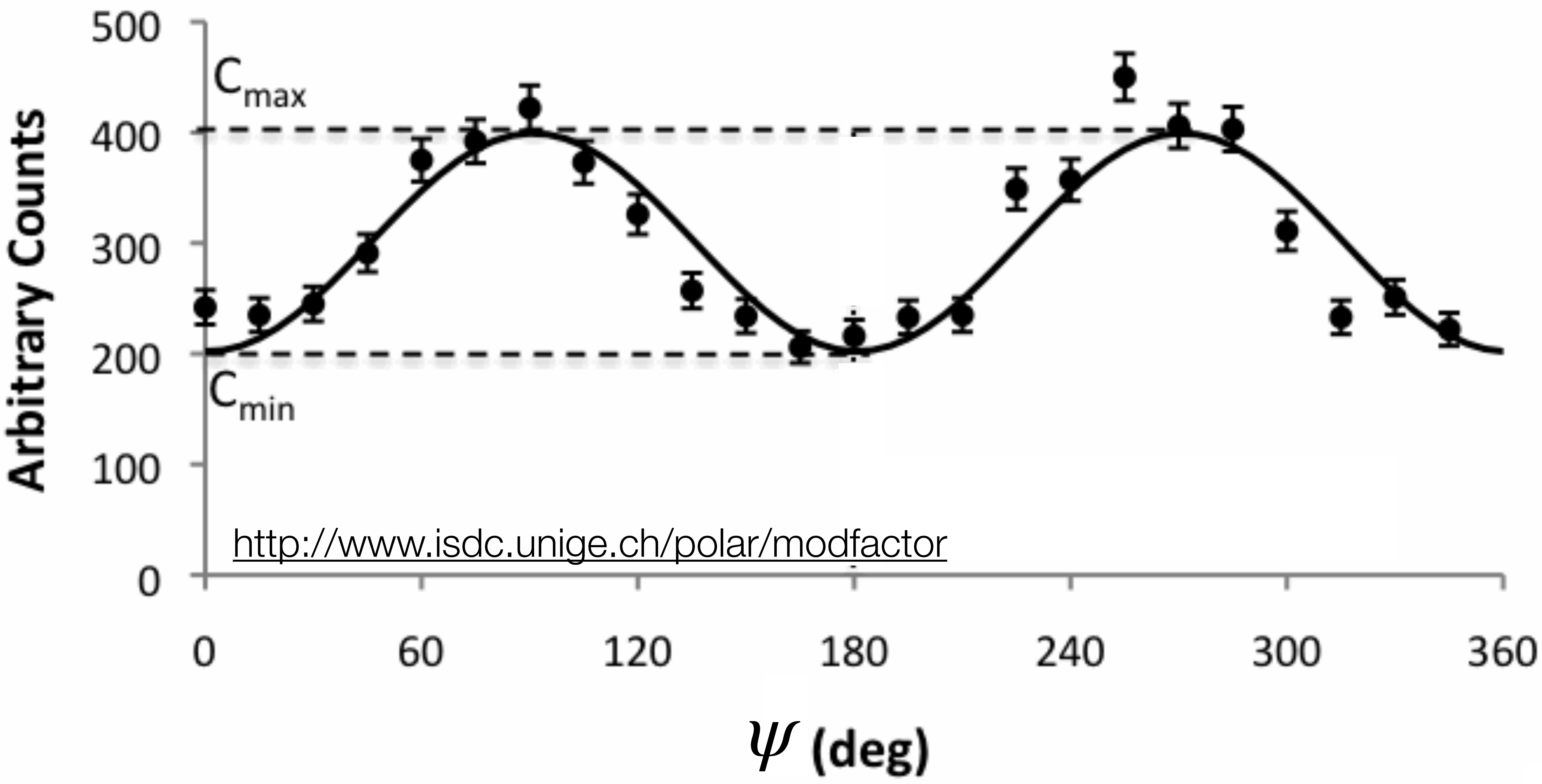


- Common alternative: Π, ϕ
  - $\Pi = (Q^2 + U^2)^{1/2} / I$
  - $\phi = \tan^{-1}(U/Q) = 2 \times$ EVPA
- A beam is "unpolarized" if the photon **set** is randomly polarized ($\Pi = V = 0$)
- MDP = 'Minimum Detectable Polarization' (at 99% conf.) = $\dfrac{4.292\sqrt{N_S + N_B}}{\mu N_S}$

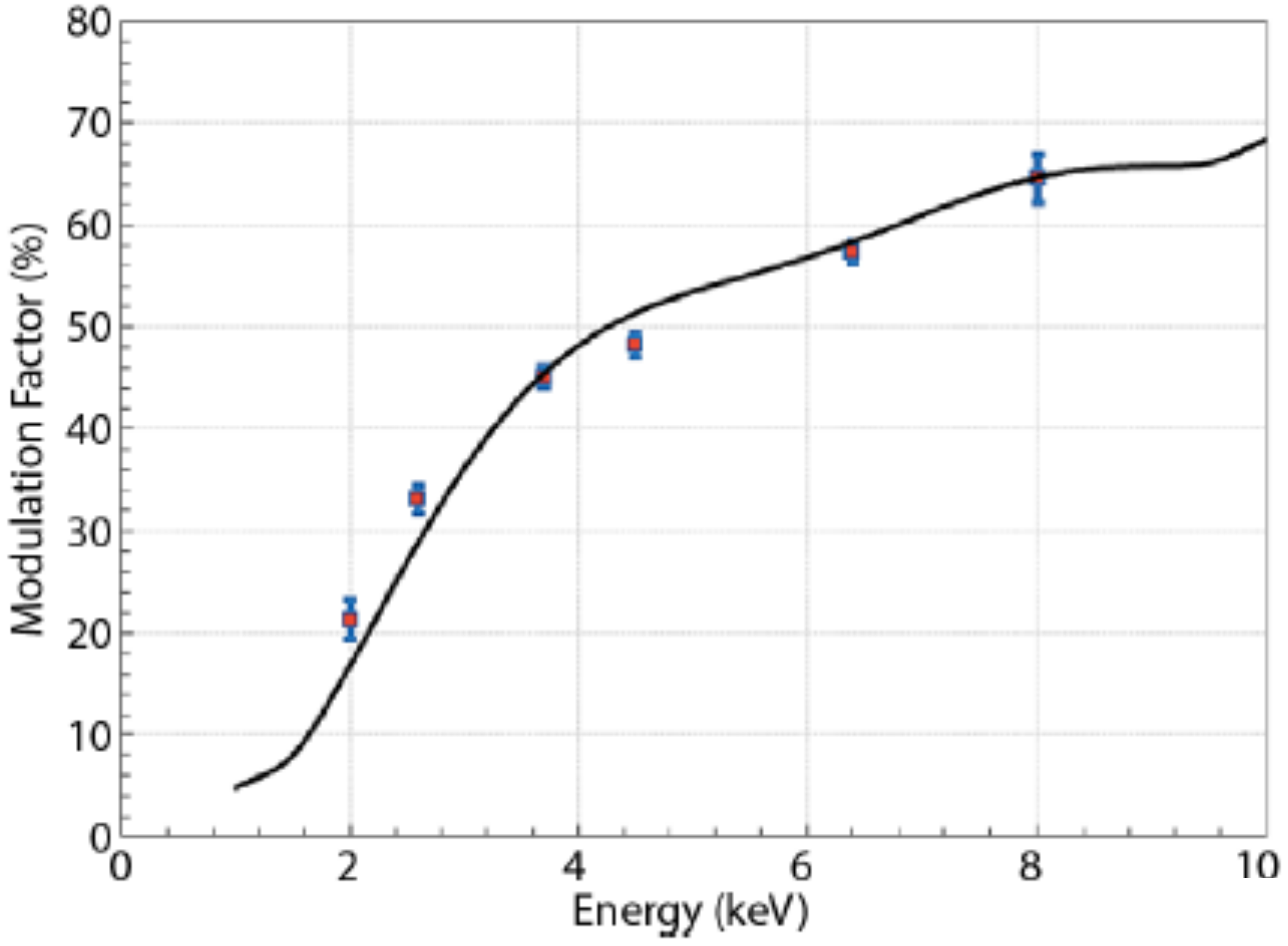
$$4.292 = 2(-\log[0.01])^{1/2}$$

Modulation of Polarized Signals

Modulation Curve 100% polarized source



IXPE Modulation Curve



Modulation Factor = $\mu = (C_{max}-C_{min})/(C_{max}+C_{min})$

$$f(\psi) = \frac{1}{2\pi}\left(1 + p_0\,\mu\cos(2(\psi-\psi_0))\right)$$

Relevant Work

- Elsner, O'Dell, & Weisskopf (2012): Gaussian statistics, BG $\mathrm{MDP}_{99} = \frac{4.292\sqrt{N_S + N_B}}{\mu N_S}$
- Kislat+ (2015): Unbinned analysis, event weighting
- ★ Strohmayer (2017): Fitting IQU spectra in xspec, mRMF $= \mu R(E; E')$
- Burgess+ (2019): Likelihood method for GRB polarimetry
- Peirson+ (2021): Machine learning to get better μ
- Marshall (2021, 2022): Likelihood method, modeling μ

} See 12/1/20 CalStats WG presentation

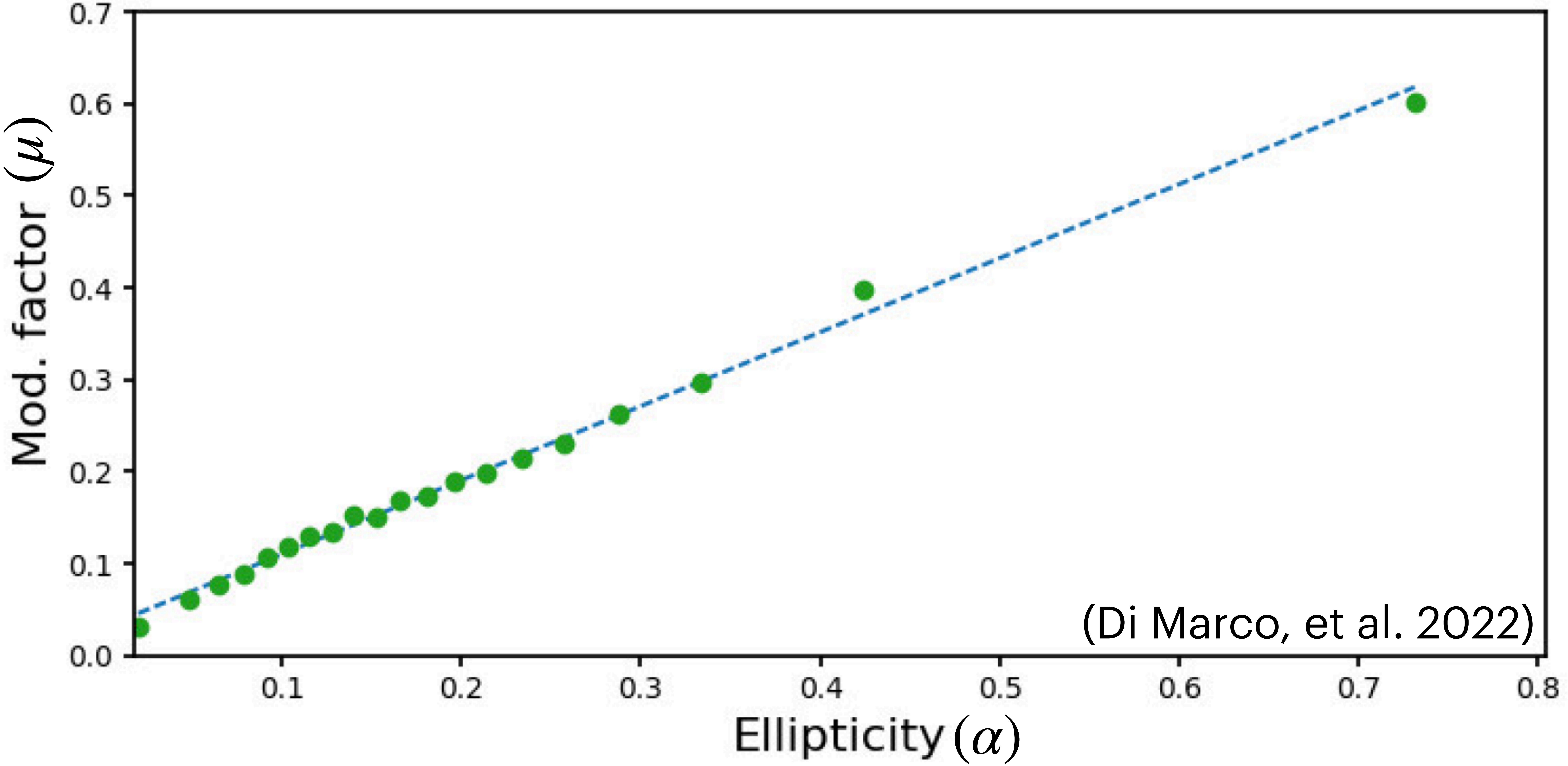
- ★ Di Marco+ (2022): Event weights using IXPE track ellipticities α
- Gonzales-Caniulef+ (2022): Likelihood method for pulsars
- ★ Marshall (in prep.): Likelihood with BG, nonuniform ψ, mRMF(E, α; E')

Likelihood Formulation (Marshall 2021)

See 12/1/20 CalStats WG presentation

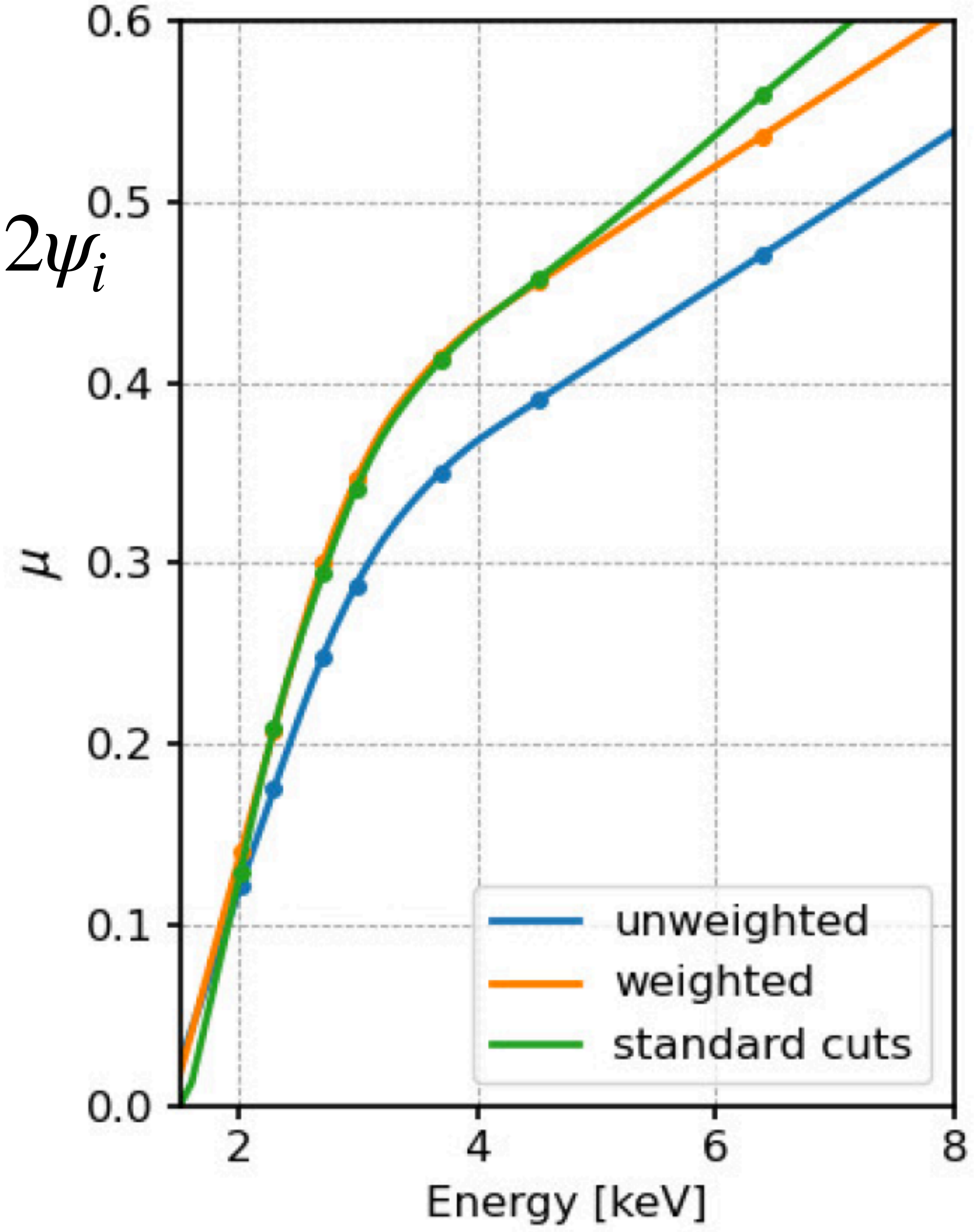
- Expected counts in $dEd\psi$ in time *T* for $Q = qI, U = uI$:
  - $\lambda(E,\psi;n_0,q,u)dEd\psi = [1+\mu_E(q\cos 2\psi + u\sin 2\psi)]n_E A_E T dEd\psi$, where
    - $A_E = A\alpha(E)$ is the instrument effective area (independent of q or u by definition)
    - $n_E = n_0\phi(E)$ has units of ph/cm^2/s/keV per unit (measured) phase angle, ψ
  - Require $\Pi^2 \equiv q^2 + u^2 \leq 1$ physically (Π is fractional linear polarization)
  - Define $\phi_0 = \tan^{-1}(u/q) = 2\varphi$
- Likelihood: $S(n_0,q,u) = -2\ln\mathscr{L} = -2\sum_i \ln\lambda(E_i,\psi_i) + 2T\int f_E A_E dE \int_0^{2\pi}[1+\mu(E)(q\cos 2\psi + u\sin 2\psi)]d\psi$ or $\tilde{S}(q,u) = -2\sum_i \ln(1+q\mu_i\cos 2\psi_i + u\mu_i\sin 2\psi_i)$
- MDP$_{99}$ $= 4.29/\sqrt{\sum \mu_E^2 C(E)}$ for small $\Pi\mu$

Inferring the Modulation Factor



IXPE Ellipticity Weighting (Di Marco+ 2022)

- Compute $w_i = \alpha_i^{0.75}$ for each event
- Estimate I,Q,U (following Kislat+ '15):
  $\mathcal{I} = \sum w_i, \quad \mathcal{Q} = 2\sum w_i \cos 2\psi_i, \quad \mathcal{U} = 2\sum w_i \sin 2\psi_i$
- Compute $\hat{\Pi} = \frac{\sqrt{\mathcal{Q}^2 + \mathcal{U}^2}}{\mu \mathcal{I}}$ with uncertainty
  $\sigma_\Pi^2 \simeq \frac{2\sum w_i^2}{\mu^2 \mathcal{I}^2} = \frac{2}{\mu^2 N_{\text{eff}}} \quad (\text{for } \Pi \ll 1)$
- Develop "weighted" modulation functions
- Weighted MDPs are ~5% better than standard



How do we fit IXPE data?

- Split spectra into I, Q, U
- E is "observed", E' is "true" and unknown
- Need RMFs, $R(E', E)$. and ARFs, $\epsilon(E')$
- Assumes mRMF = $\mu(E')R(E', E)$

- Complication: $\mu = f(\alpha)$, α = ellipticity (Di Marco+ 2022)
- Suggestion: mRMF for J (=3-10) values of α_j
  $\mathscr{M}_j(E', E) = \mu(\alpha_j, E')\epsilon(E')\phi(\alpha_j, E')R(E', E)$

$$I(E) = \int_{E'} F(E')\epsilon(E')R(E', E)dE' \tag{25}$$

$$U(E) = \int_{E'} W(E')\mu(E')\epsilon(E')R(E', E)dE' \tag{26}$$

$$Q(E) = \int_{E'} Z(E')\mu(E')\epsilon(E')R(E', E)dE' \; . \tag{27}$$

$$O(E, \psi) = I(E) + U(E)\sin(2\psi) + Q(E)\cos(2\psi) \; . \tag{28}$$

One model spectrum, $F(E')$, is folded through the full detector response function, $\epsilon(E')R(E', E)$, and the two new spectra, $W(E') = F(E')a(E')\sin(2\psi_0'(E'))$ and $Z(E') = F(E')a(E')\cos(2\psi_0'(E'))$ are folded through the "modulated response" function, $\mu(E')\epsilon(E')R(E', E)$.

Strohmayer (2017)

Suggest: Updating XSPEC analysis

- New model is $Q_j(E,\Theta) = T\int A(E')\mathcal{Q}(E',\Theta)\mathcal{M}_j(E',E)dE', \quad U_j(E,\Theta) = T\int A(E')\mathcal{U}(E',\Theta)\mathcal{M}_j(E',E)dE'$
  - Index j refers to specific values of α_j
  - New detector mRMF is $\mathcal{M}_j(E',E) = \mu(\alpha_j,E')\epsilon(E')\phi(\alpha_j,E')R(E',E)$ New!
  - where $\sum_j \phi(\alpha_j,E') = 1$ and $\sum_j \mu(\alpha_j,E')\phi(\alpha_j,E') = \mu(E')$ (unweighted, uncut)
- Original: $\lambda(n_0,\Pi,\varphi;E,\psi) = [1+\Pi\mu_E\cos(2\psi+2\varphi)]n(E')A(E')TdE'd\psi$
  - gives MDP99 $= 4.29/\sqrt{\sum \mu_{E_i}^2 C(E_i)}$
- Then $\lambda(n_0,\Pi,\varphi;E,\alpha_j,\psi) = \int dE'[1+\Pi\mathcal{M}_j(E',E)\cos(2\psi+2\varphi)]n(E')A(E')Td\psi$
- and $\tilde{S}(q,u) = -2\sum_i \ln(1+q\mu(\alpha_i,E_i)\cos 2\psi_i + u\mu(\alpha_i,E_i)\sin 2\psi_i)$

Systematic Errors, Generally

- Some methods exist for handling systematic errors
  - They’re not easy or standardized
  - There’s a scientific risk if systematics are not accounted
  - There’s a scientific risk if fixes impact model parameters
- Need a handle on “IXPE-only” systematic errors (for high SNR cases)
  - If mild, we don’t introduce corrections, apply them to other instruments
  - If not, then test for covariance of adjustments with polarization
  - If not enough, then MC trials and difficult methods needed

Systematics and polarimetry

- For $q \equiv Q/I, u \equiv U/I$ and $q, u \neq f(E)$:

$$-2\ln\mathscr{L} = S = -2\sum_{i=1}^{N}\ln[N_0(1 + q\mu_i c_i + u\mu_i s_i) + \zeta N_B] + 2N_0,$$ where

Depends only on modulation factor — not affected by systematic errors in A(E)

$c_i = \cos 2\psi_i$, $s_i = \sin 2\psi_i$, $\zeta = \Omega_S/\Omega_B$, N (N_B) is count in src (bg) region, N_0 is the expected source counts in the src region, μ_i is modulation factor for event i

- Xspec approach:

 (Strohmayer (2017)

$$I(E) = \int_{E'} F(E')\epsilon(E')R(E', E)dE'$$

$$U(E) = \int_{E'} W(E')\mu(E')\epsilon(E')R(E', E)dE'$$

$$Q(E) = \int_{E'} Z(E')\mu(E')\epsilon(E')R(E', E)dE'\ .$$

$$O(E, \psi) = I(E) + U(E)\sin(2\psi) + Q(E)\cos(2\psi)\ .$$

Note:

$$q(E) = Q(E)/I(E) = \frac{\int Z(E')\epsilon(E')R(E', E)\mu(E')dE'}{\int F(E')\epsilon(E')R(E', E)dE'}$$

$$= q\frac{\int F(E')\epsilon(E')R(E', E)\mu(E')dE'}{\int F(E')\epsilon(E')R(E', E)dE'}$$

if $Z(E) = qF(E)$

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Scenarios of Systematic Errors

- Assume $\tilde{\epsilon}(E) = \epsilon(E)/(1 + \xi[E])$ is true area, $\tilde{I}(E), \tilde{Q}(E)$ are "true" spectra, $I(E), Q(E)$ are estimated spectra using $\epsilon(E)$, $\xi \ll 1$
- Assume correct, (narrow) Gaussian RMFs
  - IXPE data only, narrow E' range: $\hat{q} = \tilde{q}$ exactly
  - External $\tilde{I}(E)$: $\hat{q} = Q(E)/\tilde{I}(E)/\mu(E) = \tilde{q}(1 + \xi)$
- Consider realistic RMFs: Gaussians with low E tails
  - If $\mu \neq f(E)$, then $\hat{q} = \tilde{q}$ regardless of $\xi(E)$ ($\alpha = 1$ in the example)
  - high E: $\hat{q} = \tilde{q}$, as low E' has no high E' tail
  - low E: more complicated, depends on spectrum
    - Soft source ($\beta \ll 1$): $\hat{q} \approx \tilde{q}[1 + \beta(\alpha - 1)(\xi_2 - \xi_1)]$
    - Hard source ($\beta \gg 1$): $\hat{q} \approx \tilde{q}[1 + \xi_2\beta(\alpha - 1)]$ — Problem!
- Note: $\hat{q}/\tilde{q} = \hat{u}/\tilde{u}$, so $\hat{\Pi}/\tilde{\Pi} = \hat{q}/\tilde{q}$ but EVPAs are **not** affected
- If $\mu(E)$ is erroneous, $\hat{\Pi}$ is affected in all cases but **not** EVPAs

$$\tilde{Q}(E)/\tilde{I}(E) = \tilde{q}\frac{\int F(E')\tilde{\epsilon}(E')R(E',E)\mu(E')dE'}{\int F(E')\tilde{\epsilon}(E')R(E',E)dE'}$$

$$\hat{q}\mu(E) = Q(E)/I(E) \equiv C_\mu(E)/C(E)$$

Define

$$E_2' > E_1', \mu_2 = \alpha\mu_1, C_2 = \beta C_1, C_{\mu,i} = \mu_i C_i, \xi_i = \xi(E_i')$$

then

$$\tilde{q} = \frac{C_{\mu,1} + C_{\mu,2}}{C_1 + C_2}, \quad \hat{q} = \frac{C_{\mu,1} + C_{\mu,2} + \xi_1 C_{\mu,1} + \xi_2 C_{\mu,2}}{C_1 + C_2 + \xi_1 C_1 + \xi_2 C_2}$$

giving $\hat{q} = \tilde{q}\left[\dfrac{1 + \frac{\xi_1 + \xi_2\beta\alpha}{1 + \beta\alpha}}{1 + \frac{\xi_1 + \xi_2\beta}{1 + \beta}}\right]$

Summary

- IXPE data are in the HEASARC, ready for analysis
- Use likelihood-based methods on unbinned data with a simple model
- Xspec-based analysis uses RMFs and ARFs for complex models
- ixpeobssim (not discussed) is highly recommended
    - can be used to simulate or analyze data
    - uses Kislat method with weights based on track ellipticities
- Improved methods are in development (one uses Neural Nets)
- Systematic errors may not affect polarization values (esp. EVPA)