

# Statistical aspects of X-ray spectral fitting



IACHEC, 13.05.2025  
Johannes Buchner

with Peter Boorman, David Homan,  
and the BXA community



# Johannes Buchner

<https://astrost.at/istics/>

Astronomy path

Chandra & XMM  
AGN surveys  
(PhD 2015)

First  
Compton-thick  
luminosity function

Interests in Postdoc:  
Black hole obscuration  
& demographics

eROSITA (2019-)  
chair of the  
AGN working group



Data science path

Bayesian inference  
with Markov Chains

pymultinest

scaling to big data &  
diverse applications  
ultranest

review of  
nested sampling  
review of  
X-ray spectral analysis

neutron stars

Astro-particles

...

exoplanets



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AUCKLAND, NEW ZEALAND



HARVARD  
UNIVERSITY



# Book chapter

## **Statistical Aspects of X-ray Spectral Analysis**

Johannes Buchner & Peter Boorman

freely available at:

<https://arxiv.org/abs/2309.05705>

includes hands-on exercises  
for both sherpa & xspec

Cosimo Bambi  
Andrea Santangelo  
*Editors*

**Handbook of  
X-ray and  
Gamma-ray  
Astrophysics**

# Workshops

## X-ray spectral fitting methods workshop



- When: 24-25. September 2019
- Where: Max Planck Institute for extraterrestrial Physics, Garching, Germany
- Who: 44 participants. Speakers: Johannes Buchner, J Michael Burgess, Joern Wilms

[VIDEO RECORDINGS, SLIDES & LINKS](#)

J Michael Burgess,  
Joern Wilms,  
JB

freely available online at <https://www.bi4best.org> including the slides and recordings.

**BiD4BESt**  
School of Astro-Statistics

Successful School of Astro-Statistics

26TH JANUARY 2021 • EVENTS, STATISTICS

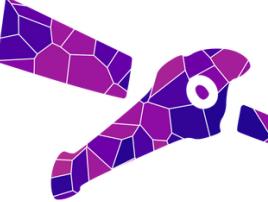
BiD4BESt has just enjoyed its first online school. Across two weeks five academics shared their knowledge on Bayesian Inference, X-ray Spectral Analysis, Machine Learning and its Applications to Astronomy, and Fitting the Spectral Energy Distributions of galaxies and AGN.



★ Holíč  
7 - 11 February 2022, Praha Czechia

Welcome to the X-ray Spectral Fitting (XSF) 2022 winter school page.

with Peter Boorman



**Chandra**  
**Data Science**

**Chandra Data Science:**  
Novel Methods in Computing and Statistics for X-ray Astronomy

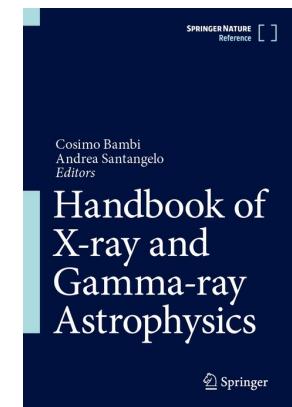
August 2021

Virtual (online)  
Hosted by the Chandra X-ray Center

“jump right to the difficult stuff and make controversial statements” – Vinay

# This talk

- book chapter contents & discussion
- Nested sampling & model comparison
- The evolving software landscape



## Disclaimer #1:

members of this group have discussed statistical aspects for a long time

Book chapter transfers knowledge from modern statistical literature to X-ray astronomy

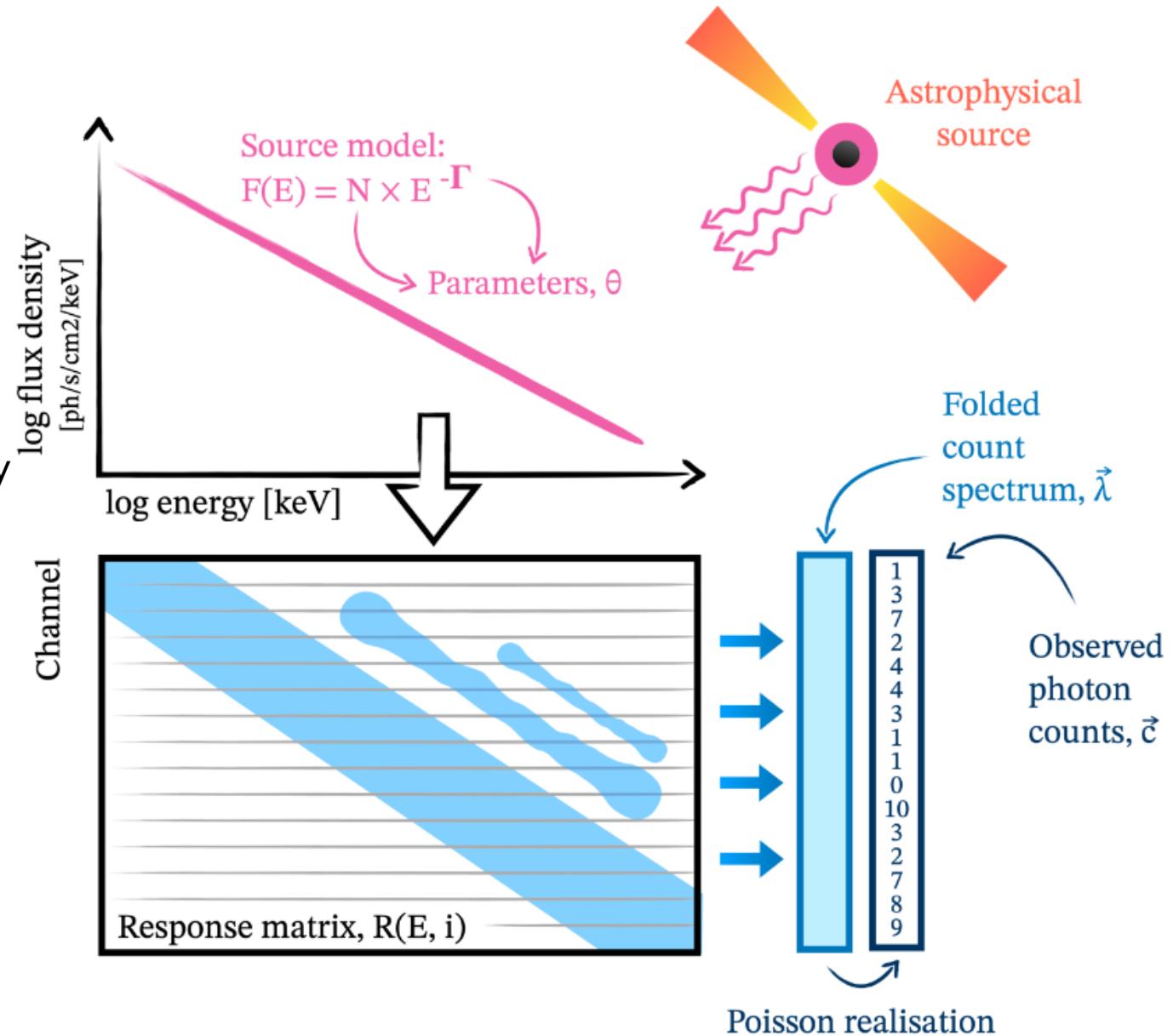
## Disclaimer #2:

unknown systematics / calibration can limit the statistical modelling

importance of IACHEC & cross-mission calibration

# Selected topics

- X-ray astronomy
  - the linear instrument model
  - Poisson statistics (cstat) & its asymmetry
  - Gaussian ( $\chi^2$ ) statistics, persistent biases
  - Background models: physical, per-bin (wstat) and its bias, empirical (with ML)
- Frequentist statistics:
  - Confidence intervals: Calibration of confidence intervals
  - Model comparison: false positive rates, false negative rates
  - Any MCMC analysis assumes a prior



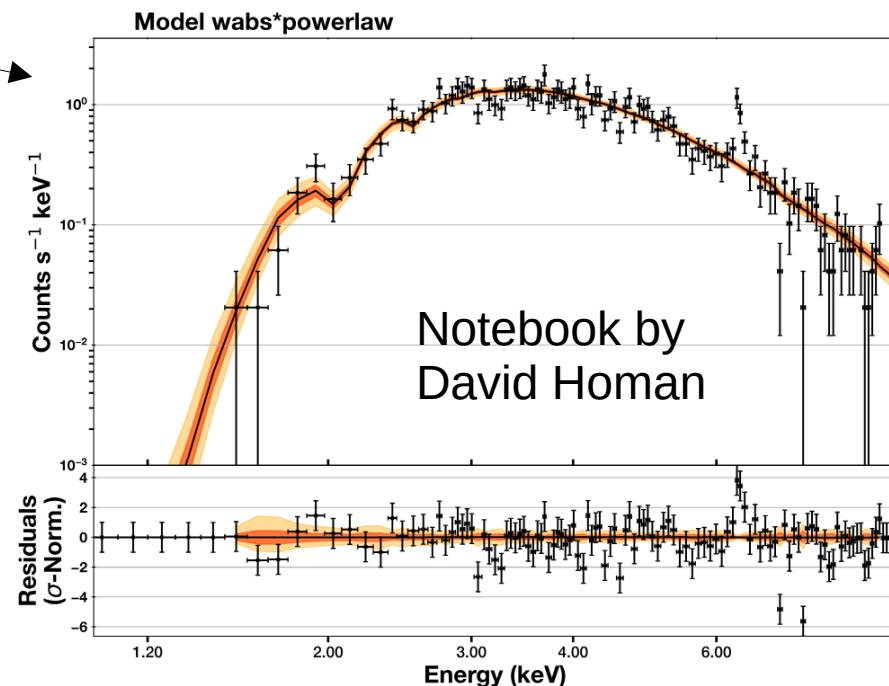
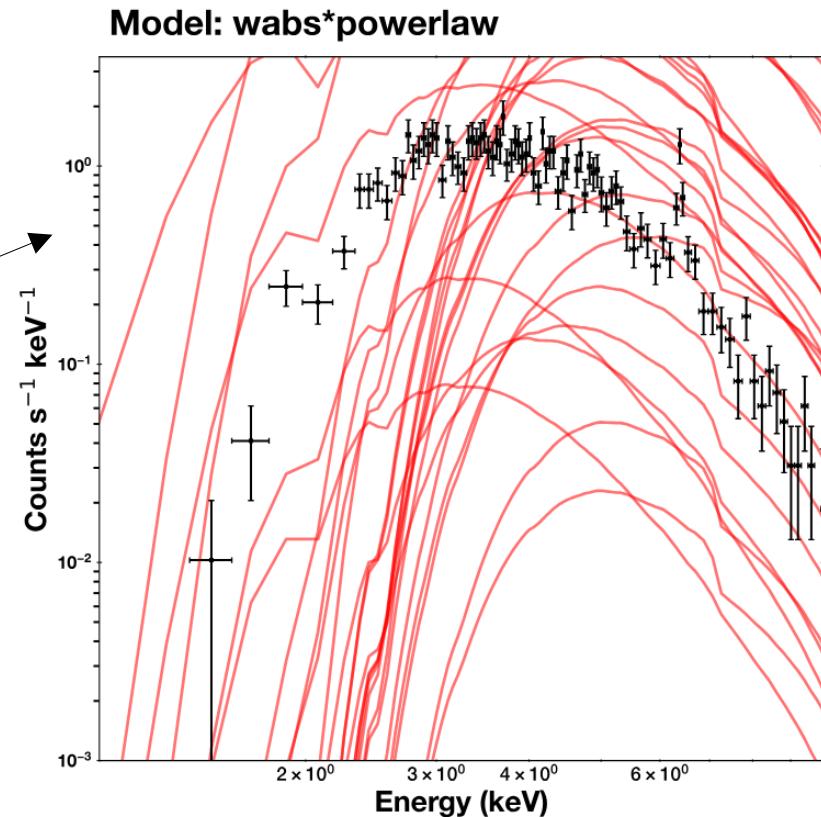
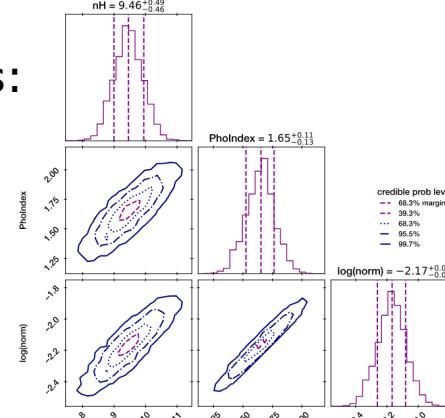
```

def cstat((N, Gamma)):
    F = N * E**Gamma
    lambda = (F * ARF) @ RMF * exposure
    poisson_lp = counts * log(lambda) + lambda
    return -poisson_lp

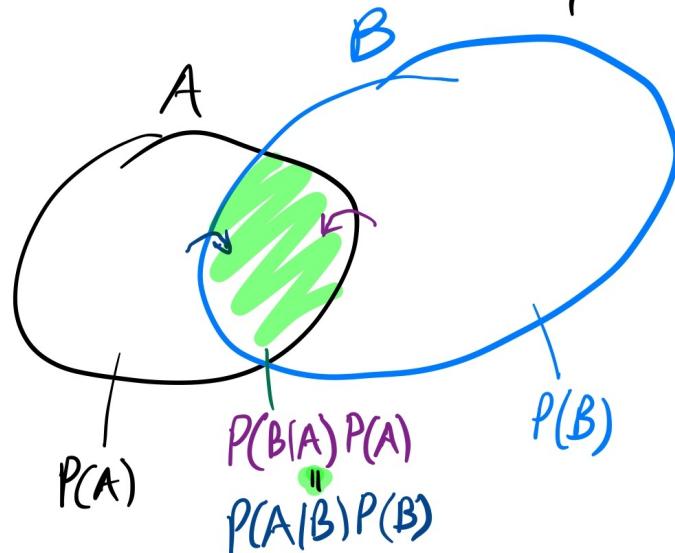
bestfit = scipy.optimize.minimize(cstat, (1, 1))
  
```

# (cont) Bayesian workflow

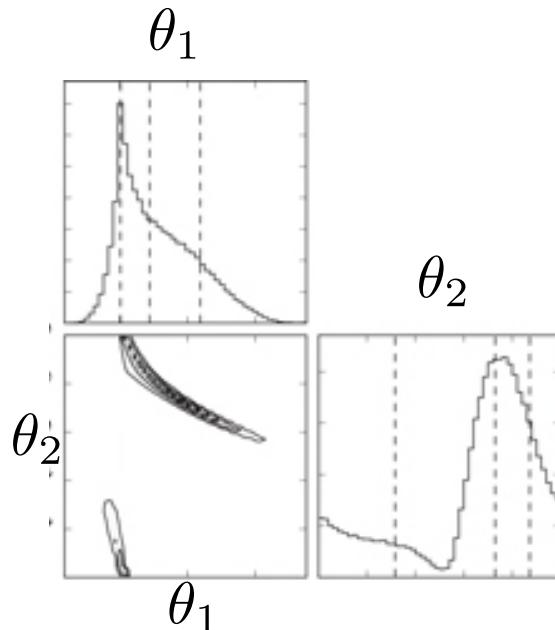
- designing priors, prior predictive checks
- Monte Carlo Algorithms:
  - MCMC: emcee, Slice sampling, HMC
  - nested sampling: MultiNest, UltraNest
- parameter posterior distributions, posterior predictive checks, quantile-quantile plots
- Bayesian model comparison: built-in Occam's razor
- Frequentist properties of Bayesian analyses
- experiment design (for proposals)
- histograms of best-fit parameters, hierarchical Bayesian models (PosteriorStacker code)



## Conditional Probability



$\theta = (L, T, \dots, \text{physical parameters})$



# Bayes 101

Bayes theorem:

Posterior distribution      Prior distribution      Likelihood function

$$P(\theta|D) = \frac{\pi(\theta) \cdot P(D|\theta)}{P(D)}$$

Where is most of the probability ?

Evidence

normalizes the posterior

“average” likelihood

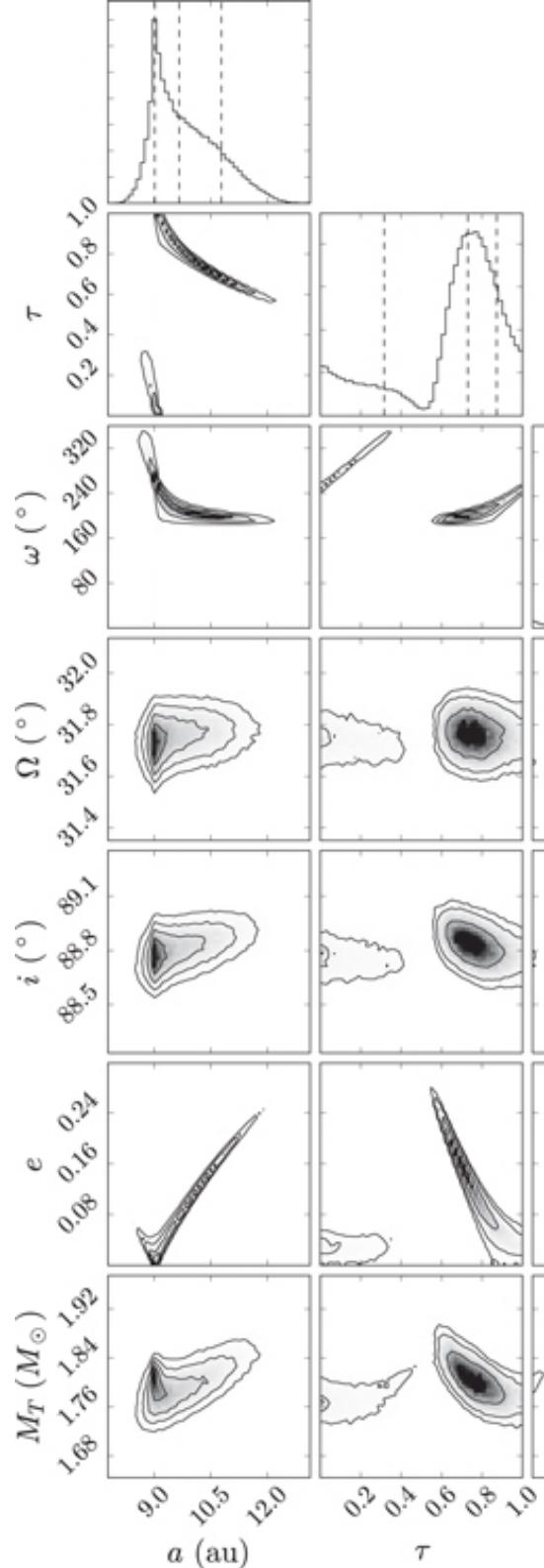
$$P(D) = \int d\theta \pi(\theta) P(D|\theta)$$

# Parameter space exploration

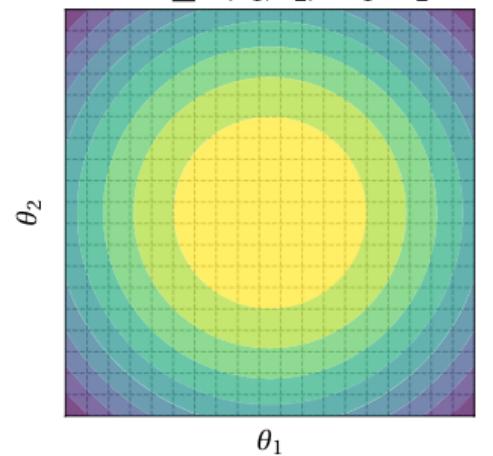
- Local optimization
  - LM, simplex, ... (many)
  - Monte carlo optimization
- Local sampling: MCMC
  - Tempering
  - Limitations

yields posterior samples
- Global optimization
  - Genetic algorithms (DE)
- Global sampling
  - Nested sampling

Yields evidence and posterior samples



$$Z \approx \sum L(\theta_1, \theta_2) \Delta\theta_1 \Delta\theta_2$$



Riemann integration

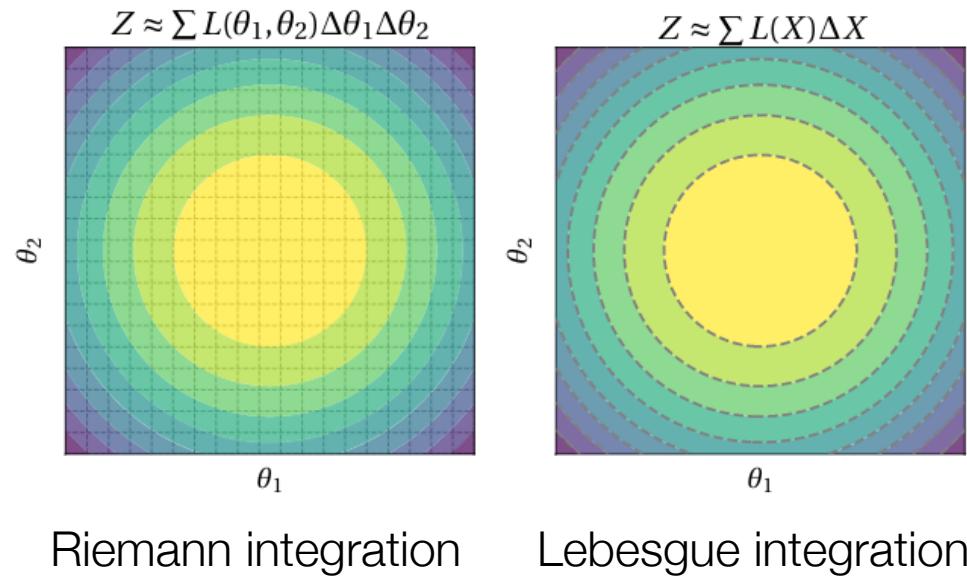
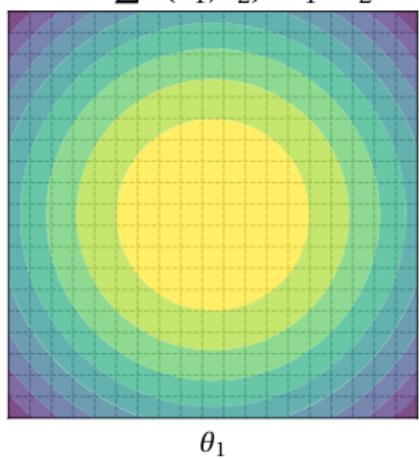


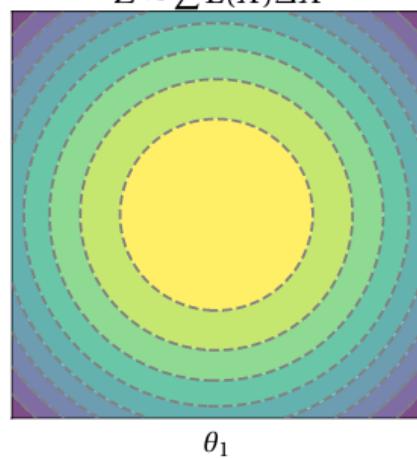
Figure 1 | Illustrations of NS algorithm.

$$Z \approx \sum L(\theta_1, \theta_2) \Delta\theta_1 \Delta\theta_2$$

 $\theta_2$ 

Riemann integration

$$Z \approx \sum L(X) \Delta X$$

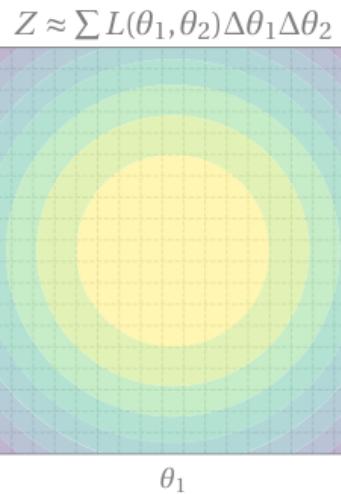
 $\theta_2$ 

Lebesgue integration

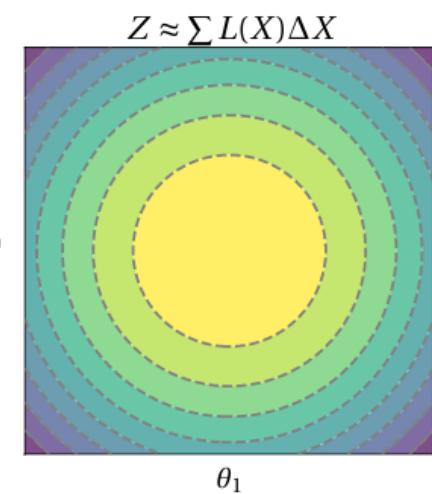
 $\mathcal{L}$ 

Volume  
shrinkage  
 $\approx 1/N$

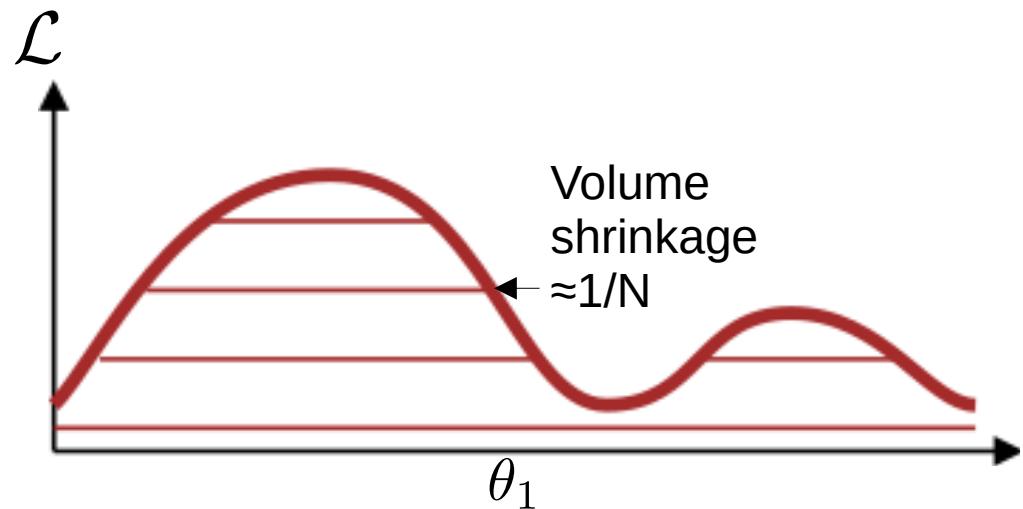
 $\theta_1$ 

 $\theta_2$  $\theta_1$ 

Riemann integration

 $\theta_2$  $\theta_1$ 

Lebesgue integration



Volume  
shrinkage  
 $\approx 1/N$

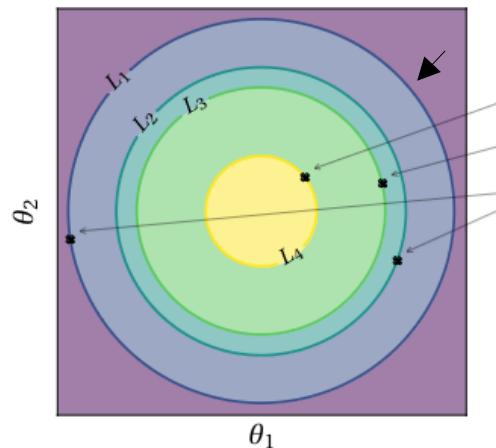
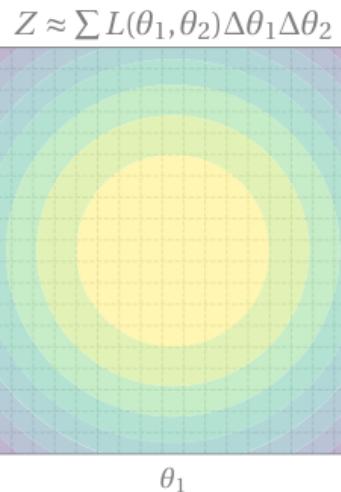
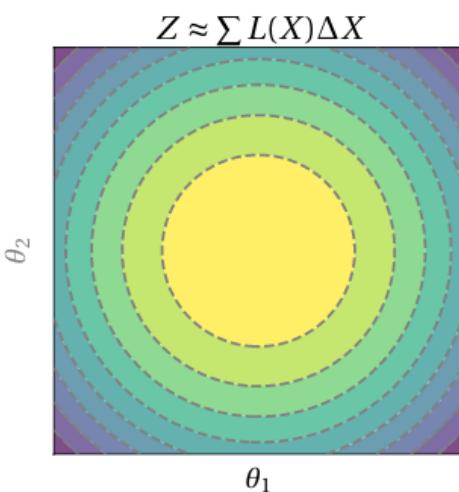


Figure 1 | Illustrations of NS algorithm.



Riemann integration



Lebesgue integration

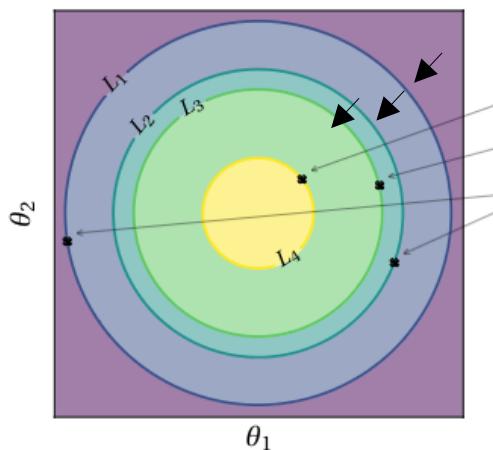
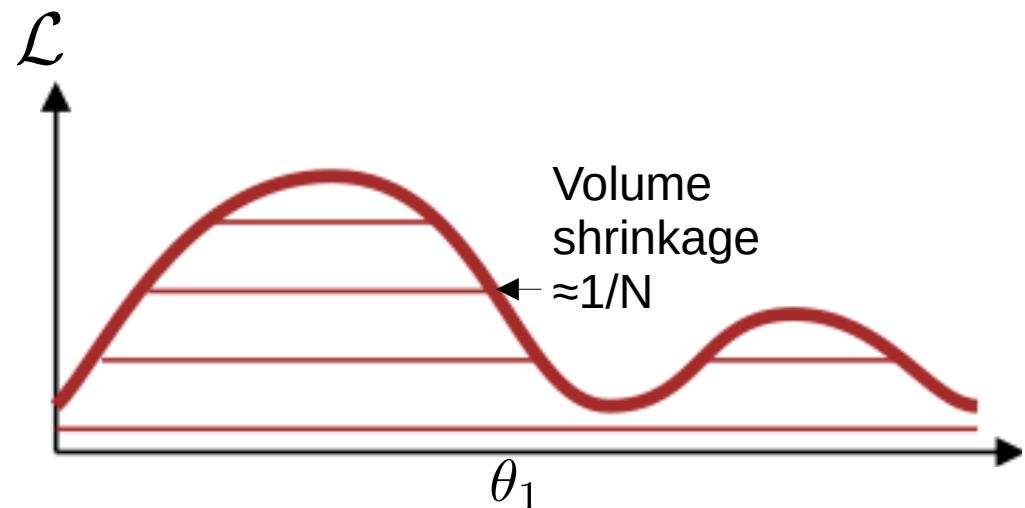
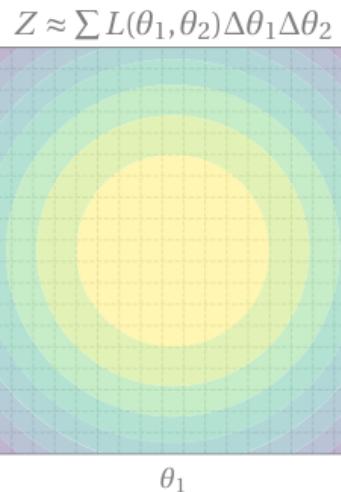
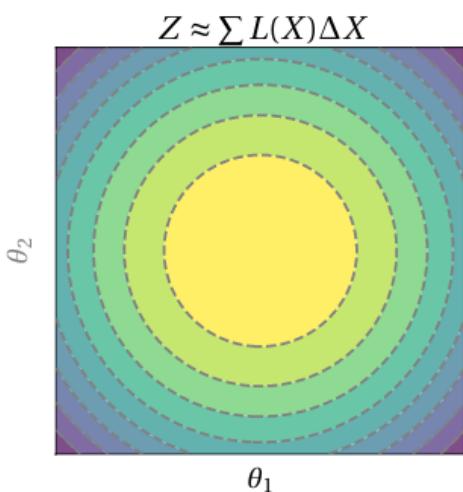


Figure 1 | Illustrations of NS algorithm.



Riemann integration



Lebesgue integration

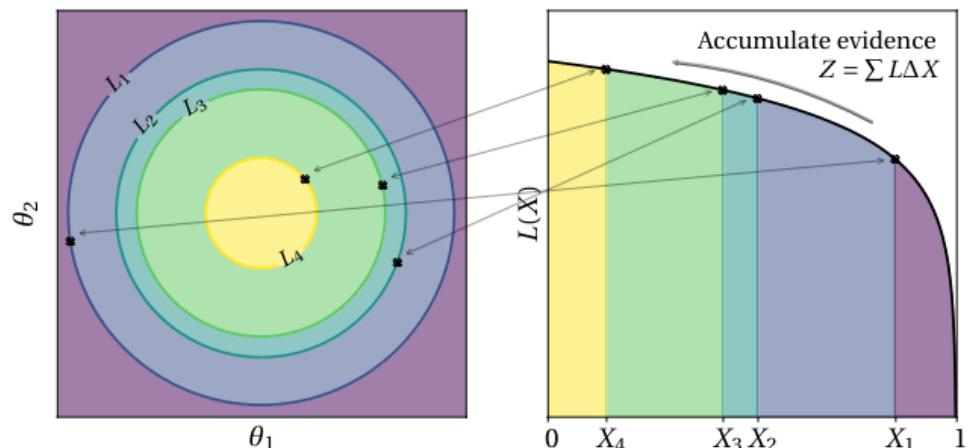
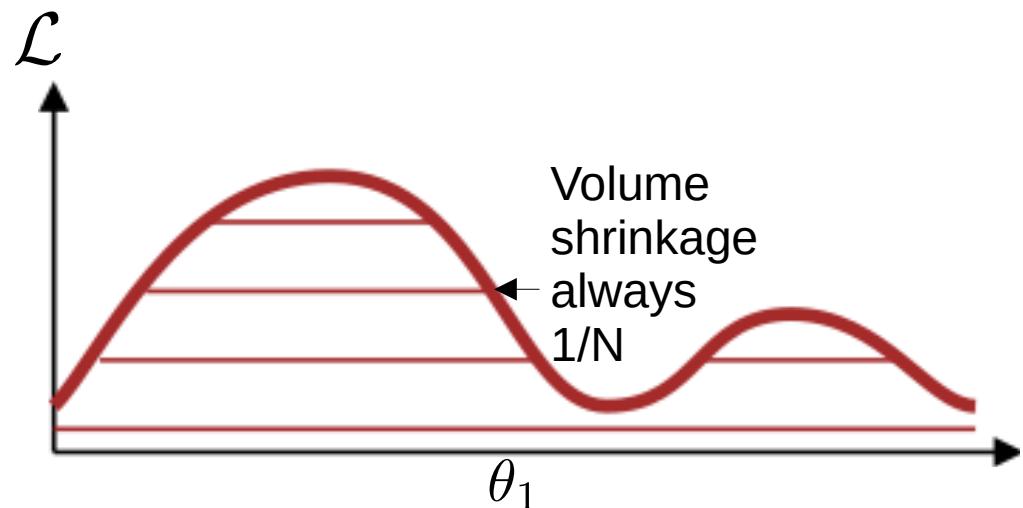
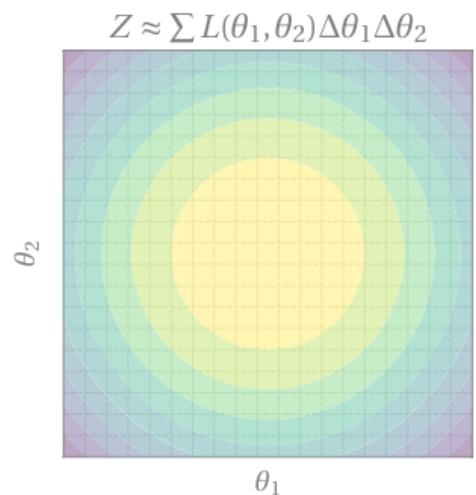
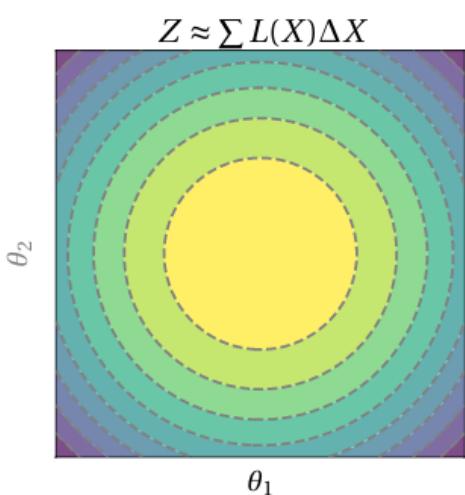


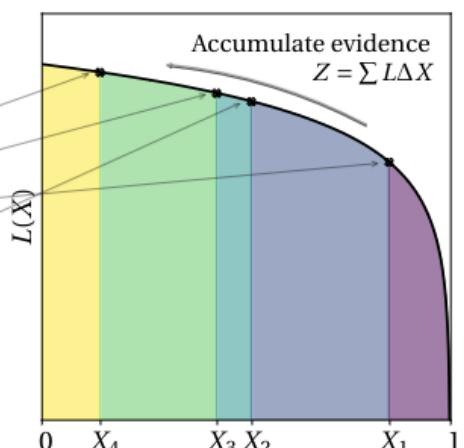
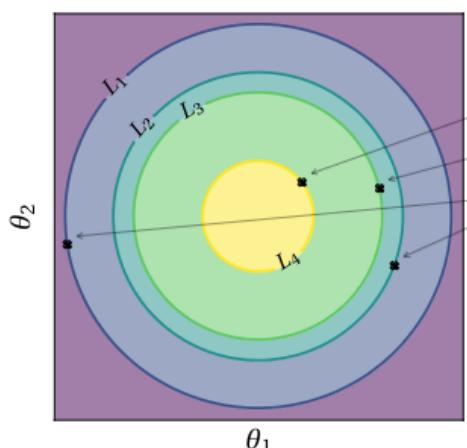
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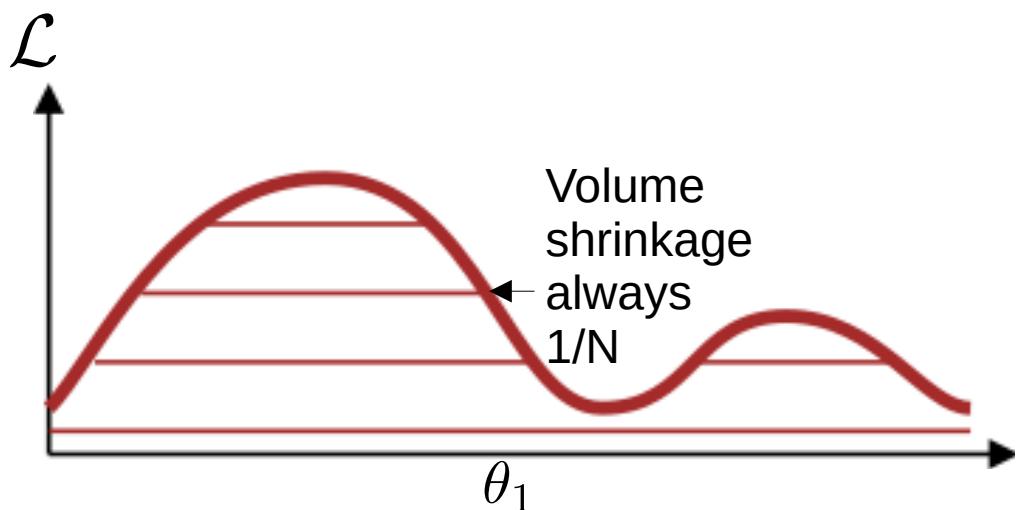
Riemann integration



Lebesgue integration



Convergence proof of  $Z$  and posterior :  
e.g. Evans (2007), Chopin&Robert (2010)



Systematic literature review  
“Nested Sampling Methods” Buchner (2023)

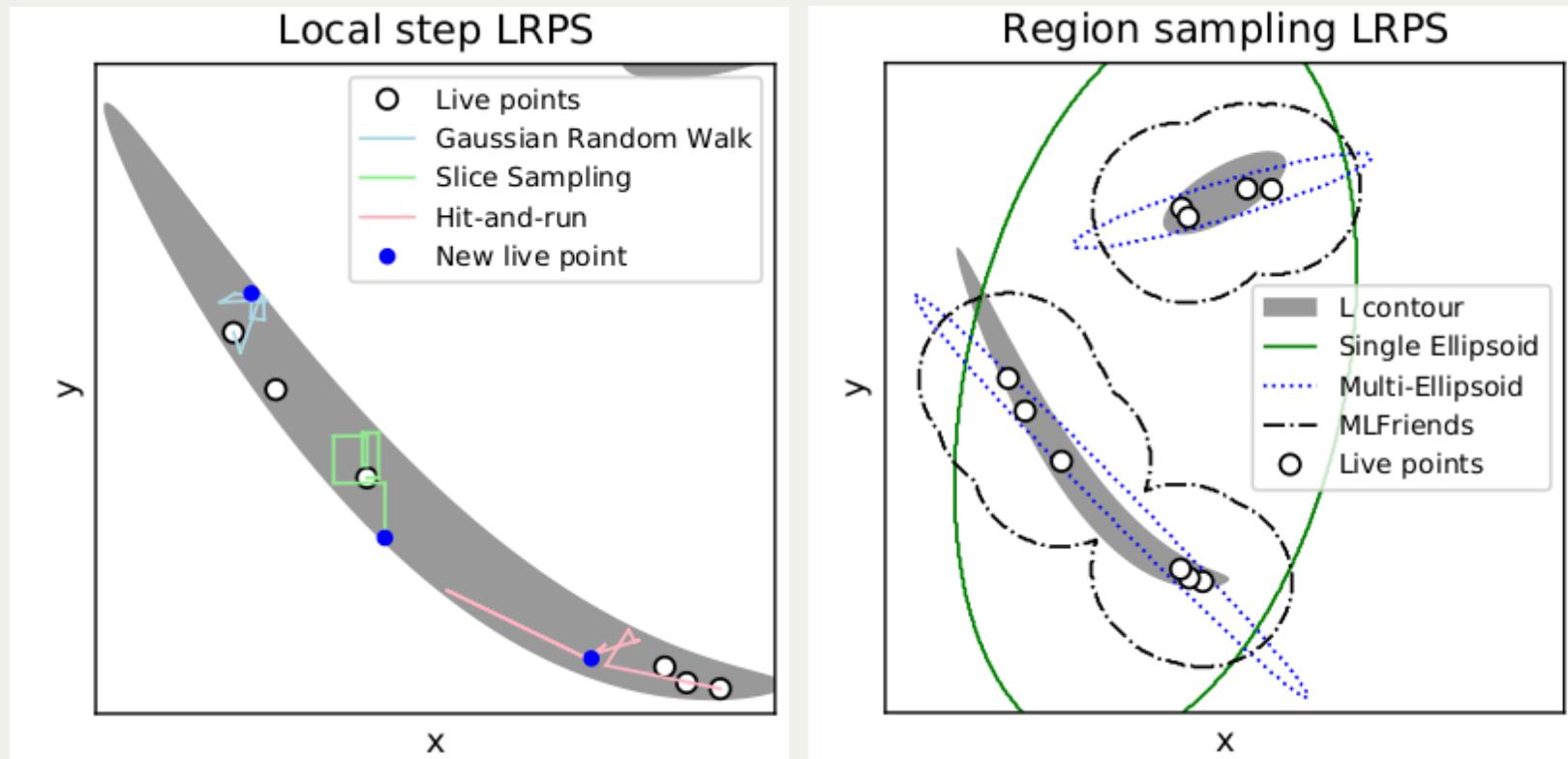
>100 papers

- Theory
- Estimators
- Termination
- Diagnostics
- variations:
  - Soft constraint
  - Variable number of live points
  - Parallelisation
- Likelihood-restricted prior sampling (LRPS)

Figure 1 | Illustrations of NS algorithm.

# Missing ingredients

- MCMC: Insert tuned transition kernel
- NS: Likelihood restricted prior sampling (LRPS)
  - General solutions: MultiNest, MCMC, HMCMC, Galilean, RadFriends (UltraNest), PolyChord



Animation:  
<https://johannesbuchner.github.io/mcmc-demo/app.html#RadFriends-NS,standard>  
(via [chifeng.github.io](http://chifeng.github.io))

# So what is BXA?

Idea: make  
physical parameter inference &  
model comparison  
easy & practical

Buchner+14 (1326 citations)

XMM2Athena  
319,565 X-ray sources  
processed (Webb+23)

parallelisation,  
resuming  
sophisticated, robust

**inference engine**

based on nested sampling

MultiNest  
UltraNest



community models  
fully-fledged  
**fitting** data formats  
**environment**  
sherpa  
pyxspec  
(threeml)  
(spex)

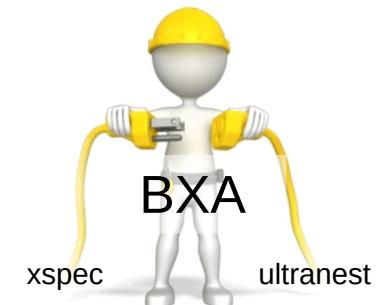
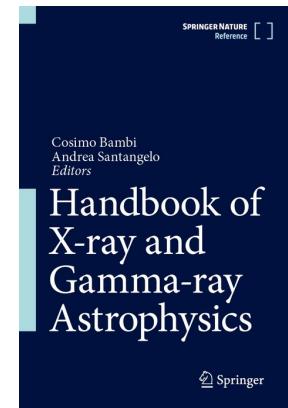
+ background models  
+ some visualisation tools

# An evolving software landscape

-   ,  ;  3ML
  - maintainance is institutional effort
  - Xspec models a community focal point
- 2014: BXA: xspec/sherpa plug-in for modern inference algorithms Buchner+14
- 2022: Model emulators Kerzendorf+22  
Matzeu+22
- 2024: Diff PPL: e.g. jaxspec
  - Require re-implementing models!Dupourqué+24  
Barret+24
- Missing? partially diff XSF

# Take-aways

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- Nested sampling & model comparison
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