A Probabilistic Method of NXB Removal for X-ray Astronomy

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Take Home Message

This is not a talk about how to estimate non x-ray background events - it is a talk about how to better remove them from your event list. The calculations and images were designed for ART-XC data, but can be applied to any high energy event list data. It is especially well suited to diffuse, low signal-to-noise regions of the sky or moving telescopes.

The Key Challenge

X-ray astronomy is almost always photon starved, and in many instances we only observe a few sky + NXB counts in a region of interest. We want to separate the sky from the background.

The simple calculation

- Observe k counts in a detector/sky region
- Expect μ_N NXB events in this region
- Interested in the expected number of sky events, μ_S
- Naively, $\mu_\mathrm{S} = \mathrm{k} - \mu_\mathrm{N}$

Limitations

Simple subtraction has limitations for X-ray data at low counts

- Subtraction can go negative - not physical
- Fluctuations can make spurious signals appear "real"
- Difference of two Poisson RV's is not a Poisson
- Cannot ID any specific event as sky/NXB.

Assumptions

- We will assume that μ_N and μ_S are Poisson processes.
- We will also assume that μ_N is known
- We observe k_N NXB counts and k_S sky photons, but only know that $k_N + k_S = k$.

The Math I

The Poisson distribution is at the center of all of these calculations

$$P(k|\mu) = \frac{e^{-\mu} \times \mu^{k}}{k!}$$

The Bayesian Perspective I

For a uniform prior on μ_S, a known value of μ_N, and an observed number of counts k we can construct the posterior PDF analytically

$$\mathcal{P}(\mu_S|\mathbf{k},\mu_N)\mathbf{d}\mu_S = \frac{\mathbf{e}^{-(\mu_N+\mu_S)}\left(\mu_S+\mu_N\right)^{\mathbf{k}}}{\Gamma_U(\mathbf{k}+1,\mu_N)}\mathbf{d}\mu_S \tag{1}$$

Maximum of this posterior PDF is at the larger of $\mu_S = \mathbf{k} - \mu_N$ and $\mu_S = 0$

The Bayesian Perspective II

For this posterior we can construct analytic formulae for mean and CDF of μ_S

$$\langle \mu_\mathsf{S} \rangle = \frac{\Gamma_\mathsf{U}(\mathbf{k}+2, \mu_\mathsf{N}) - \mu_\mathsf{N}\Gamma_\mathsf{U}(\mathbf{k}+1, \mu_\mathsf{N})}{\Gamma_\mathsf{U}(\mathbf{k}+1, \mu_\mathsf{N})} \tag{2}$$

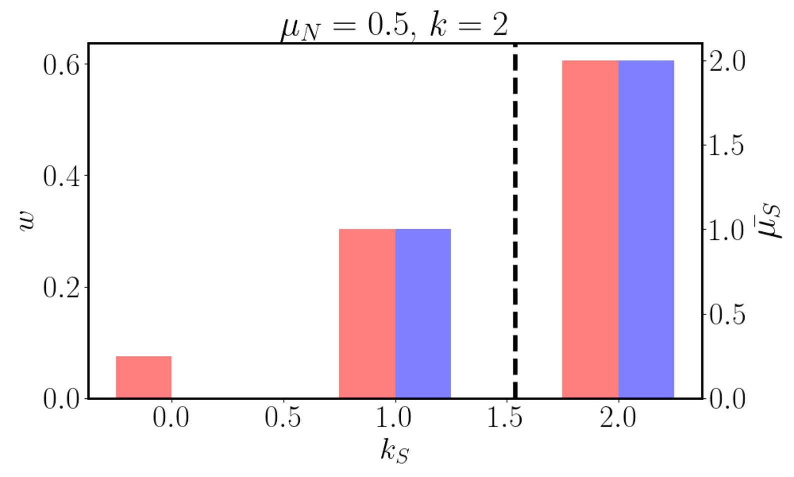
$$\int_0^\mathsf{x} \mathcal{P}(\mu_\mathsf{S}|\mathbf{k}, \mu_\mathsf{N})\mathbf{d}\mu_\mathsf{S} = \frac{\gamma_\mathsf{L}(\mathbf{k}+1, \mathbf{x}+\mu_\mathsf{N}) - \gamma_\mathsf{L}(\mathbf{k}+1, \mu_\mathsf{N})}{\Gamma_\mathsf{U}(\mathbf{k}+1, \mu_\mathsf{N})} \tag{3}$$

A Better Summary Statistic

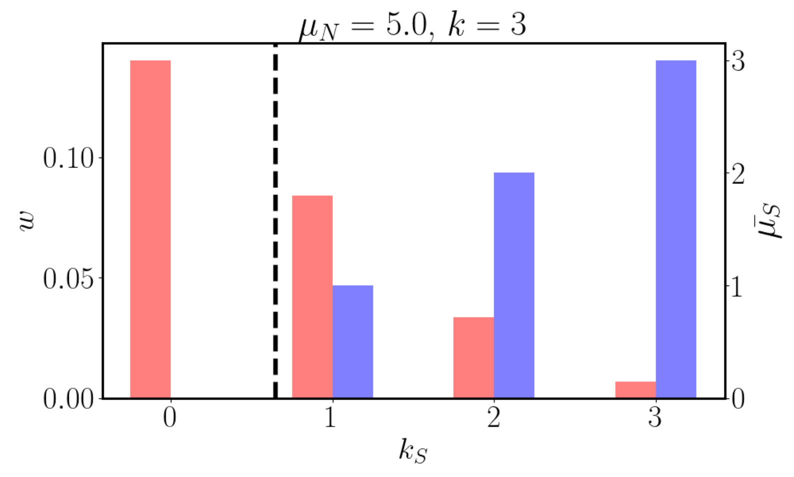
Both the mean and median of the posterior give possible summary statistics for the “net” counts. But there is an even better value that can be constructed from the micro-states.

$$\mu_{\mathsf{S}}^{\star} = \frac{1}{\mathsf{V}_1} \sum_{\mathsf{k_N}=0}^{\mathsf{k}} (\mathsf{k} - \mathsf{k_N}) \times \frac{\mathsf{e}^{-\mu_{\mathsf{N}}} \mu_{\mathsf{N}}^{\mathsf{k_N}}}{\mathsf{k_N}!}$$

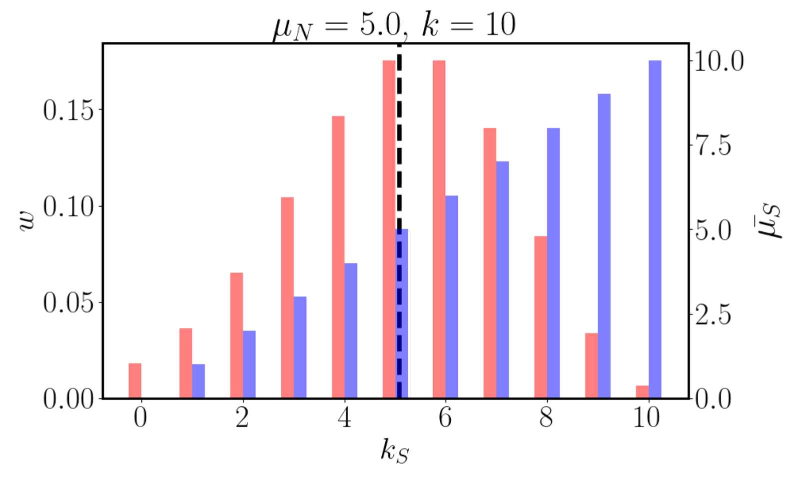
Examples



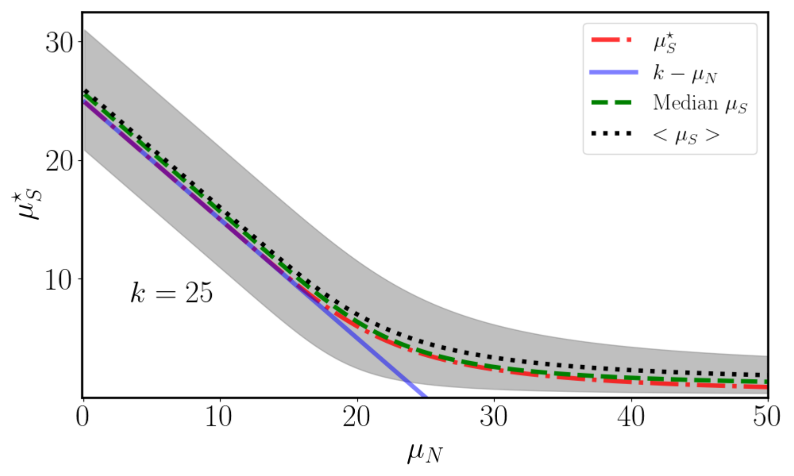
Examples



Examples



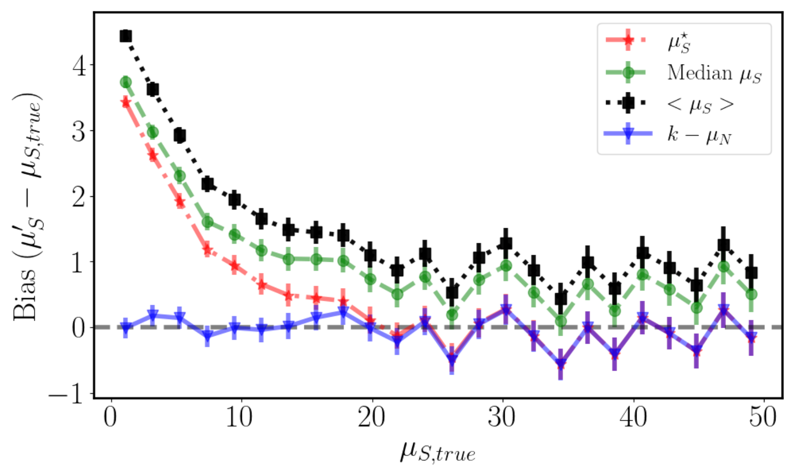
Old Versus New Calculation



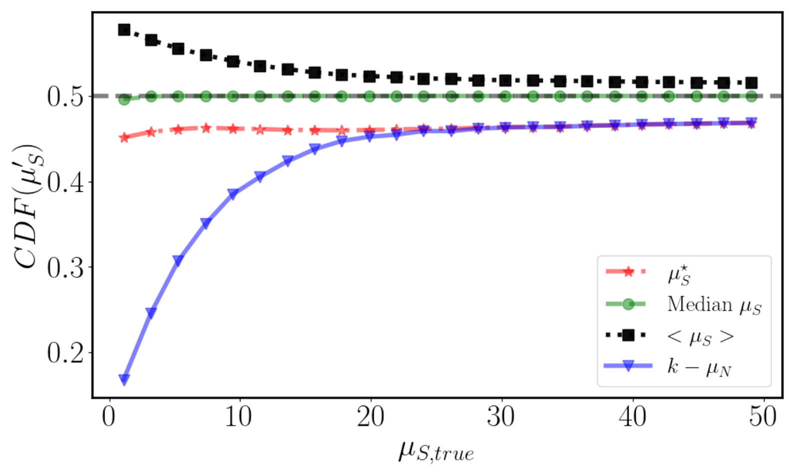
Advantages of this New Method

- Always non-negative. Only get zero net counts when $\mathsf{k} = 0$.
- Net counts converge to zero as $\mu_{\mathsf{N}} \gg \mathsf{k}$
- Converges to $\mathsf{k} - \mu_{\mathsf{N}}$ in the limit of high signal (i.e. $\mathsf{k} \gg \mu_{\mathsf{N}}$)
- Can construct event specific probabilities $\mathsf{p_S} = \frac{\mu_{\mathsf{S}}^{\star}}{\mu_{\mathsf{S}}^{\star} + \mu_{\mathsf{N}}}$.

Bias From Monte Carlo Simulations



Enclosed CDF

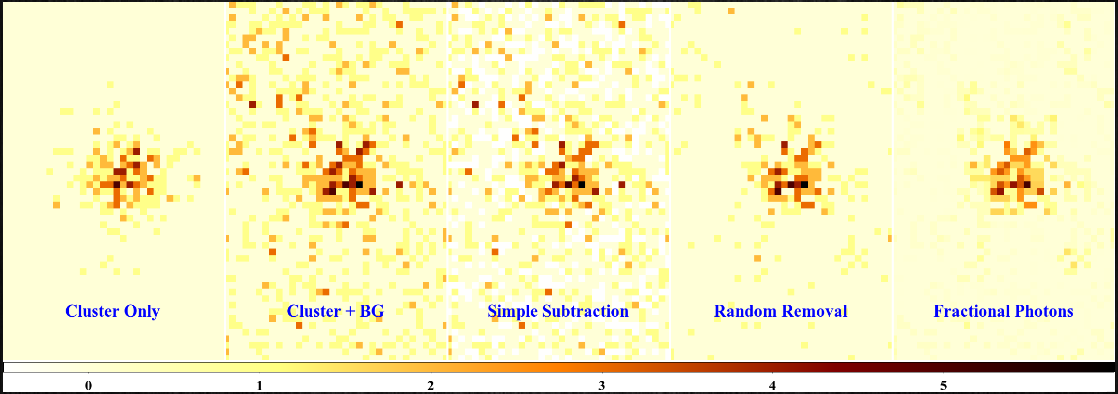


Event Specific Probabilities

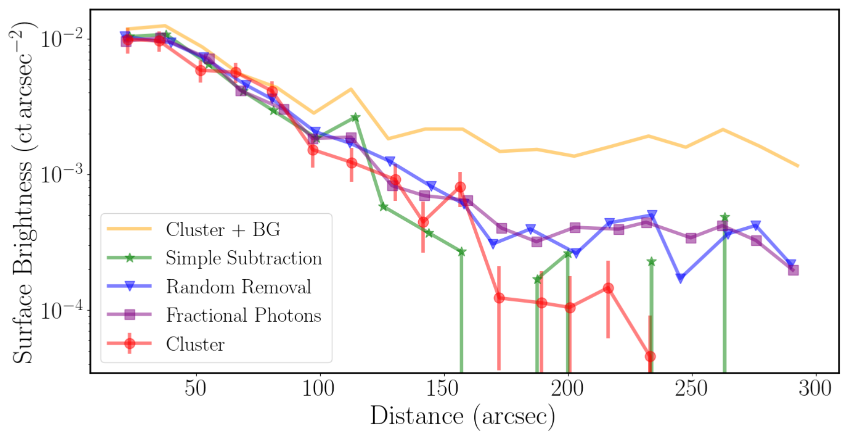
Here are three ways to use event specific probabilities (p_S) to clean images of NXB for X-ray telescopes, depending on your interests and needs:

- Random Removal: select a random number between 0 and 1 for each event. If that random number is less than p_S keep it.
- Fractional Photons: Instead of projecting 1 whole event onto the sky, only project p_S events.
- False Probability: Project $p_N = 1 - p_S$ onto the sky. As you add telescopes take the product in order to reduce noise.

Simulated Cluster Images



Simulated Cluster Profiles



Next Steps

- Paper coming to MNRAS soon - just got first referee report
- A version of this calculation for Chandra data will be integrated into CIAO, with development and science testing underway
- Develop use with other observatories
- Investigate use with other modes beyond imaging